

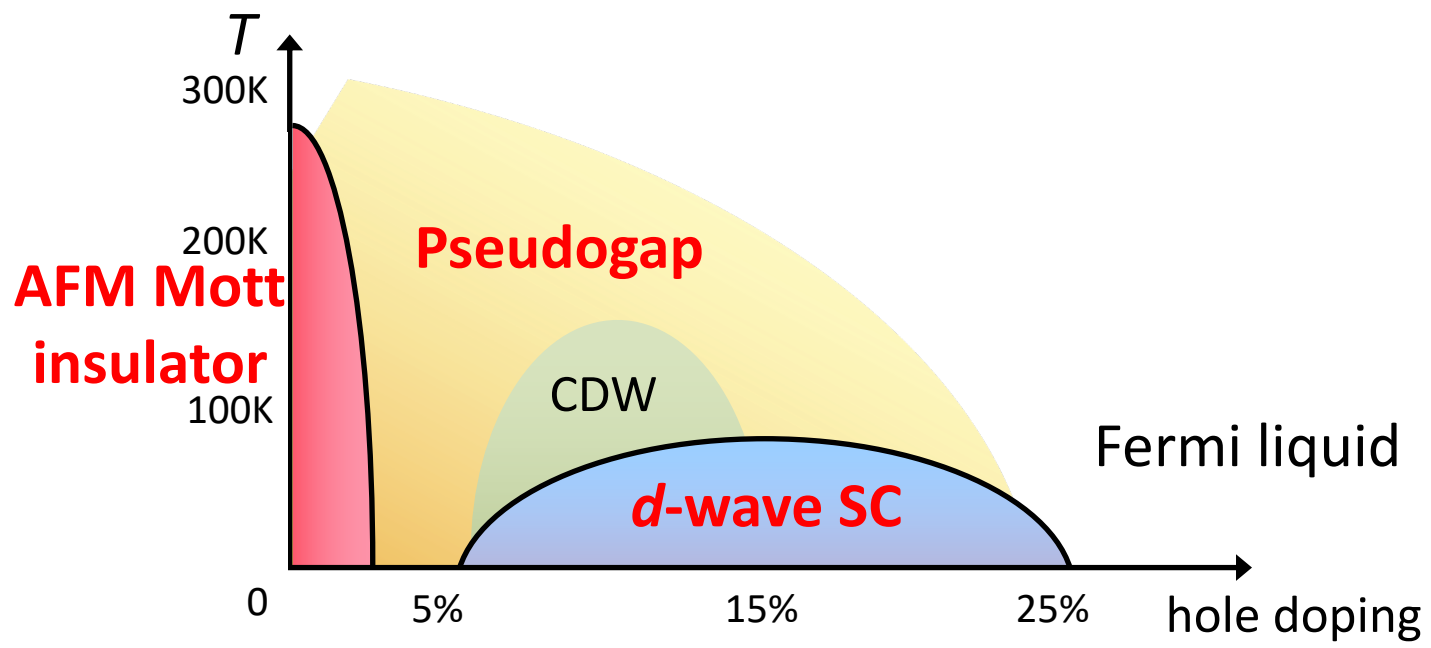
Hidden Fermionic excitation at the origin of Mott insulator, pseudogap and high-temperature superconductivity

RIKEN Center for Emergent Matter Science

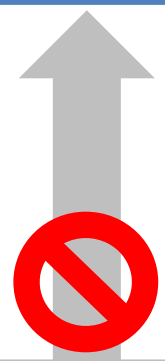
Shiro Sakai

July 18, 2019

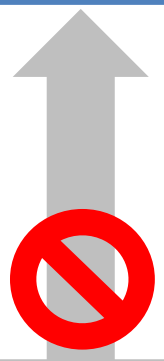
High- T_c cuprates



Mott insulator	Pseudogap state	High- T_c SC
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Band theory

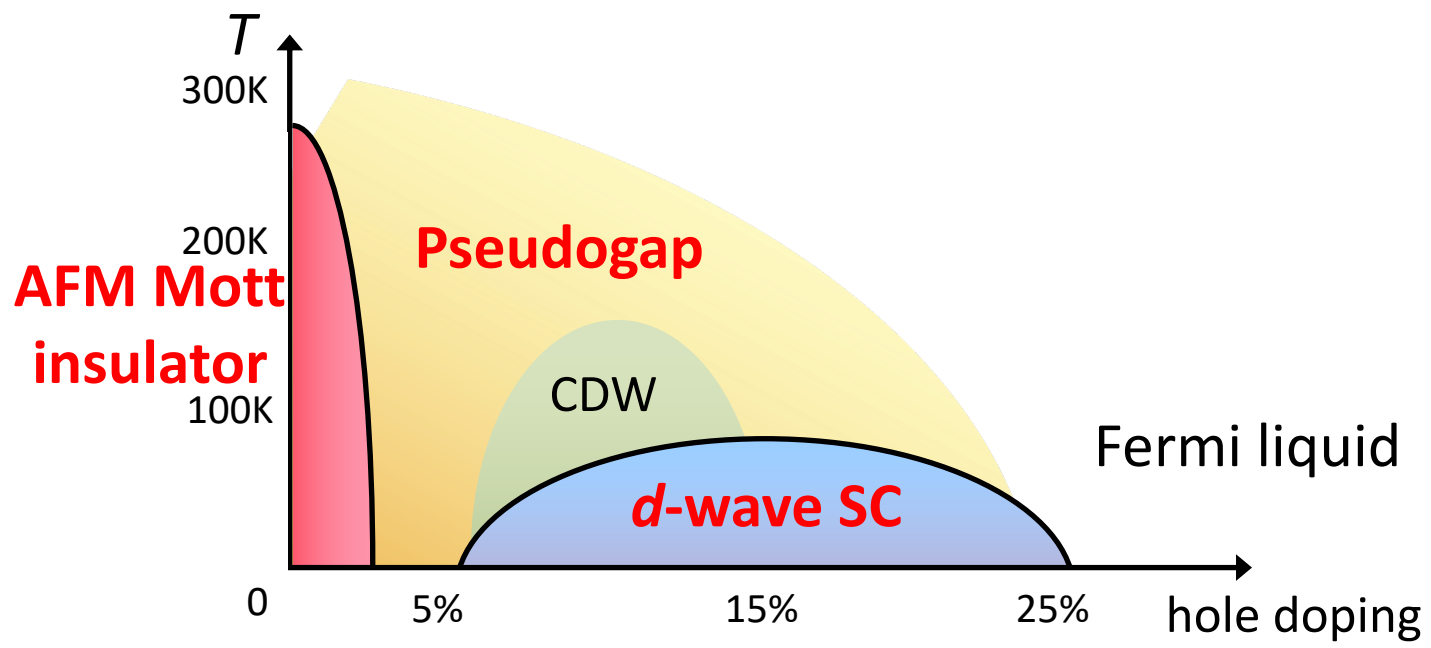


Fermi-liquid theory



BCS theory

High- T_c cuprates



Mott insulator	Pseudogap state	High- T_c SC
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*What is essential to these three states?
What is missing in these three theories?*



Band theory

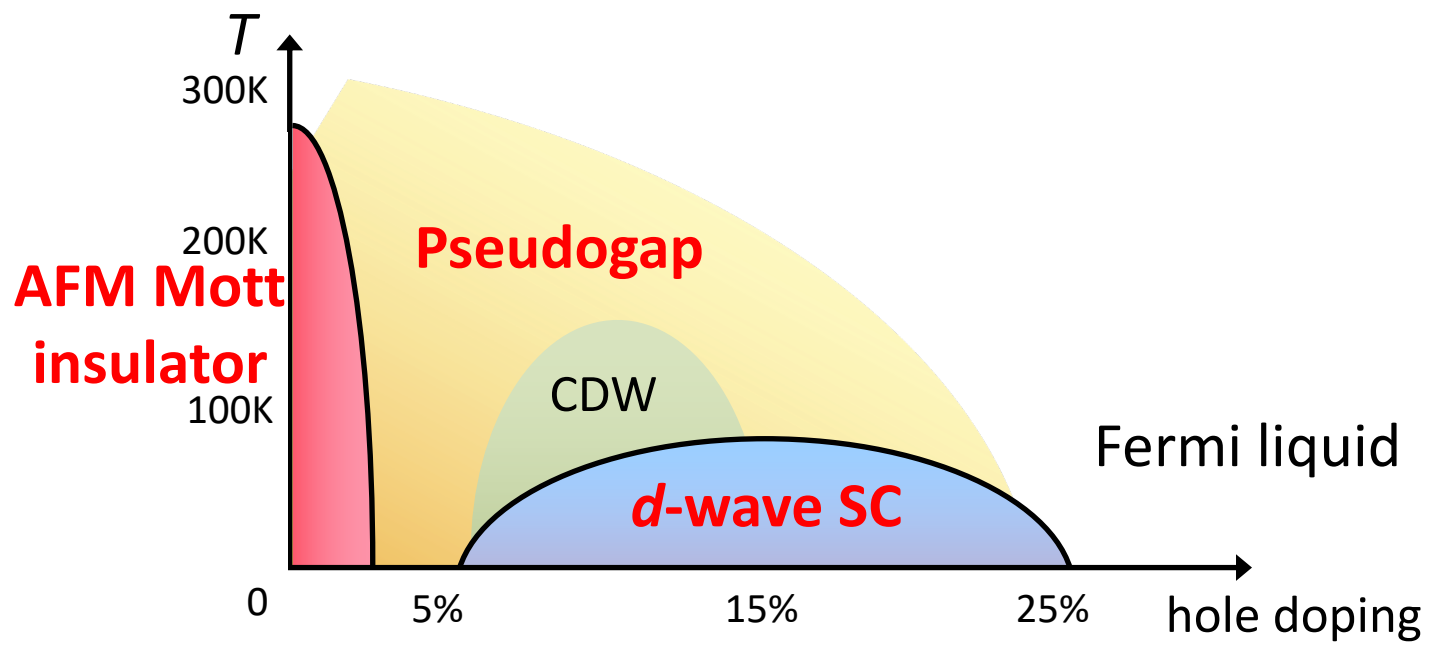


Fermi-liquid theory



BCS theory

High- T_c cuprates



Mott insulator	Pseudogap state	High- T_c SC
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Singularity of self-energy



Band theory



Fermi-liquid theory



BCS theory

Singularity (pole) of self-energy

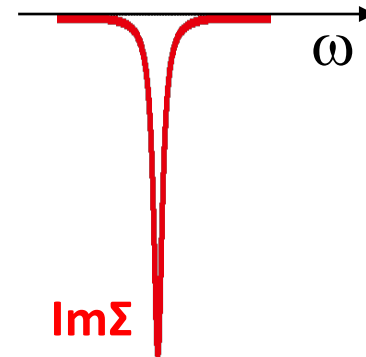
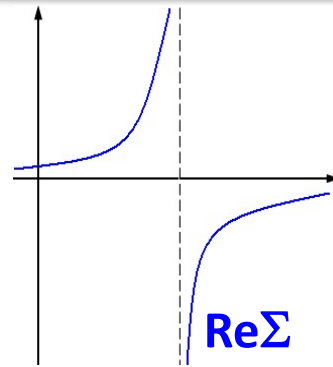
1-electron Green's function

$$G(\mathbf{k}, \omega) = \frac{1}{\omega - \varepsilon_{\mathbf{k}} - \Sigma(\mathbf{k}, \omega)}$$

ω dependence
is a key.

$\Sigma(\mathbf{k}, \omega) = \infty \iff G(\mathbf{k}, \omega) = 0$
 $A(\mathbf{k}, \omega) = 0 \iff$ No electron at (\mathbf{k}, ω)
i.e., gap.
(in normal state)

$$\Sigma \sim \frac{1}{\omega - p}$$



Need to go beyond one-particle & perturbative theories

Numerical simulations can

go beyond

Band theory
Fermi-liquid theory
BCS theory

Cluster dynamical mean-field theory

[Hettler *et al.*, PRB'98; Kotliar *et al.*, PRL'01]

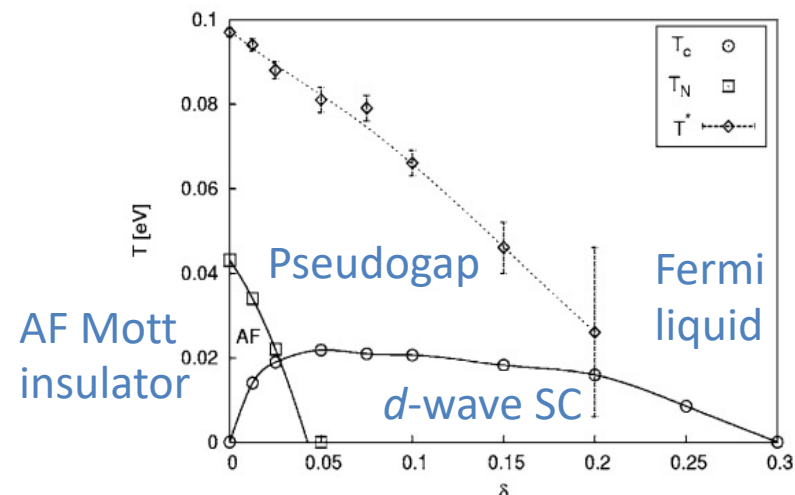
- Capable to describe
the singularity of Σ
- Full short-range correlations
- Unbiased



- Indeed reproduces cuprates' phase diagram!

Sordi *et al.*, PRL'12; Gull *et al.*, PRL'13; ...

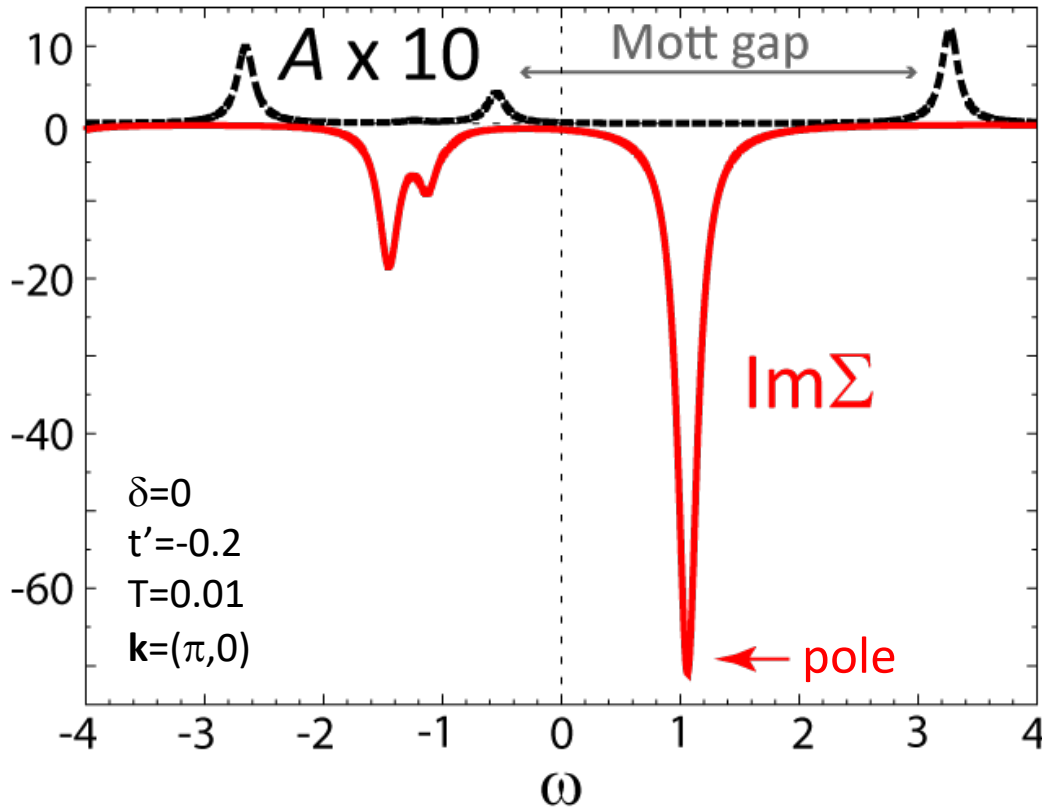
2D Hubbard model



[Maier *et al.*, RMP'05]

Self-energy pole generating the Mott gap

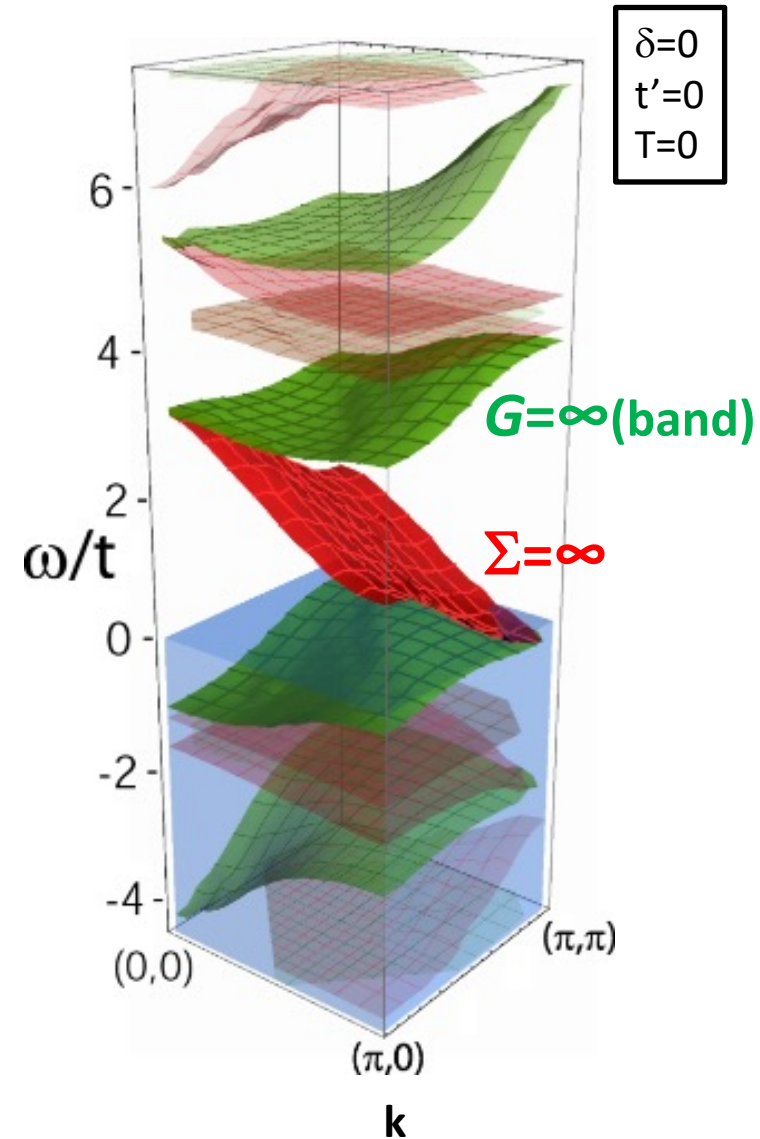
Mott insulator



**CDMFT for
2D Hubbard
 $t=1$ & $U=8$**

$$A = -\frac{1}{\pi} \text{Im}G$$

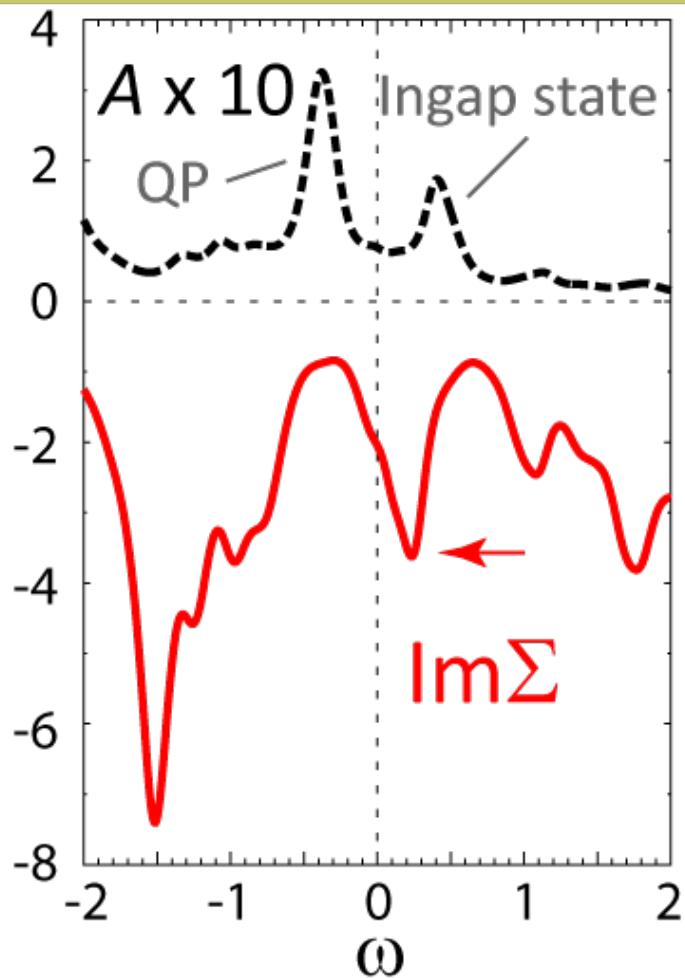
$$G = \frac{1}{\omega - \varepsilon_k - \Sigma}$$



SS, Y. Motome and M. Imada,
PRL102, 056404 (2009).

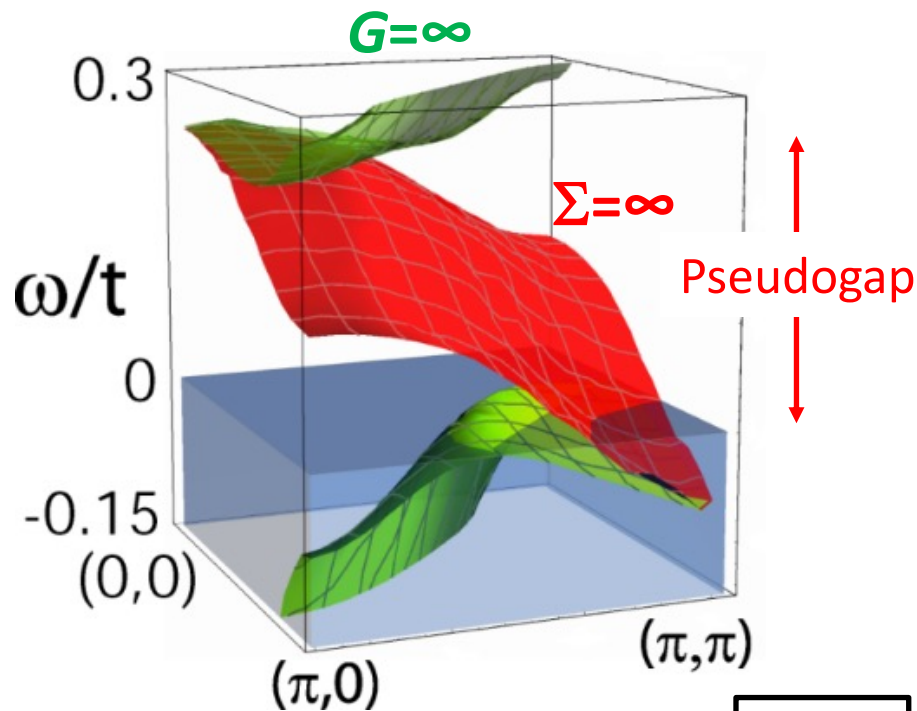
Self-energy pole generating the pseudogap

Pseudogap



$\delta=0.05$
 $t'=-0.2$
 $T=0.06$
 $\mathbf{k}=(\pi,0)$

Normal-state solution at $T=0$

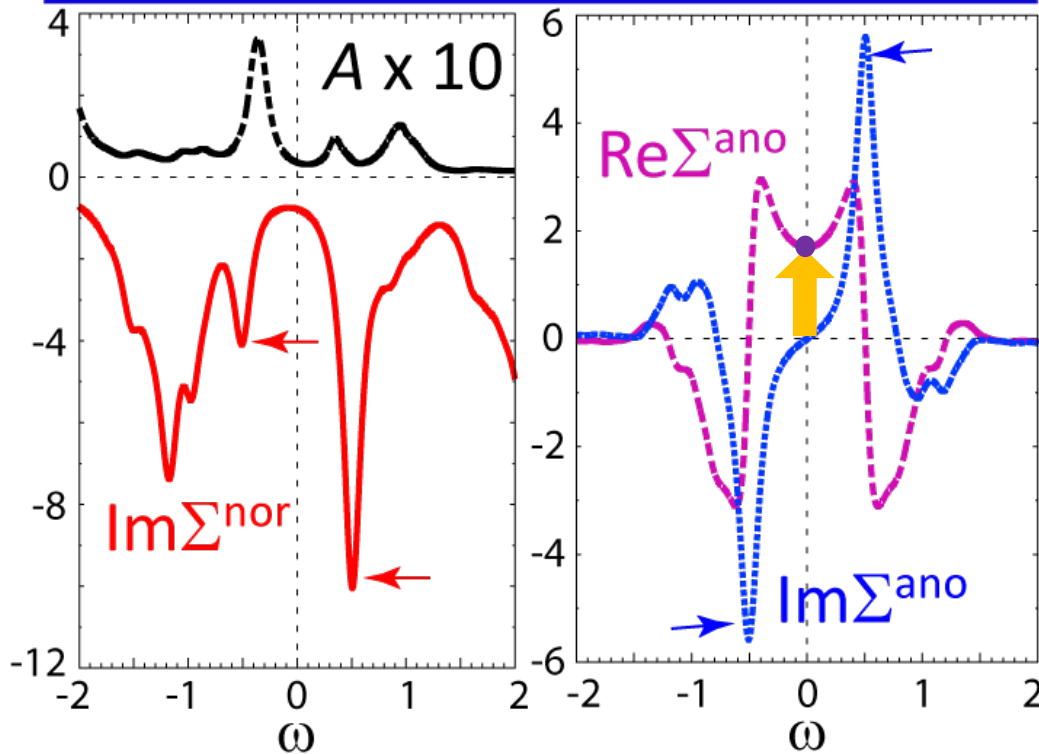


$\delta=0.09$
 $t'=0$
 $T=0$

SS, Y. Motome and M. Imada,
 PRL102, 056404 (2009).

Self-energy pole making the SC “high- T_c ”

Superconductor



$\delta=0.05$
 $t'=-0.2$
 $T=0.01$
 $\mathbf{k}=(\pi,0)$

$$\hat{\Sigma} = \begin{pmatrix} \Sigma^{\text{nor}} & \Sigma^{\text{ano}} \\ \Sigma^{\text{ano}} & -\Sigma^{\text{nor}}(-\omega)^* \end{pmatrix}$$

Both Σ^{nor} and Σ^{ano} have poles at the same ω .

$$\text{Re}\Sigma^{\text{ano}}(\mathbf{k}, \omega = 0) = \frac{2}{\pi} \int_0^\infty \frac{\text{Im}\Sigma^{\text{ano}}(\mathbf{k}, \omega')}{\omega'} d\omega' \quad (\text{Kramers-Kronig relation})$$

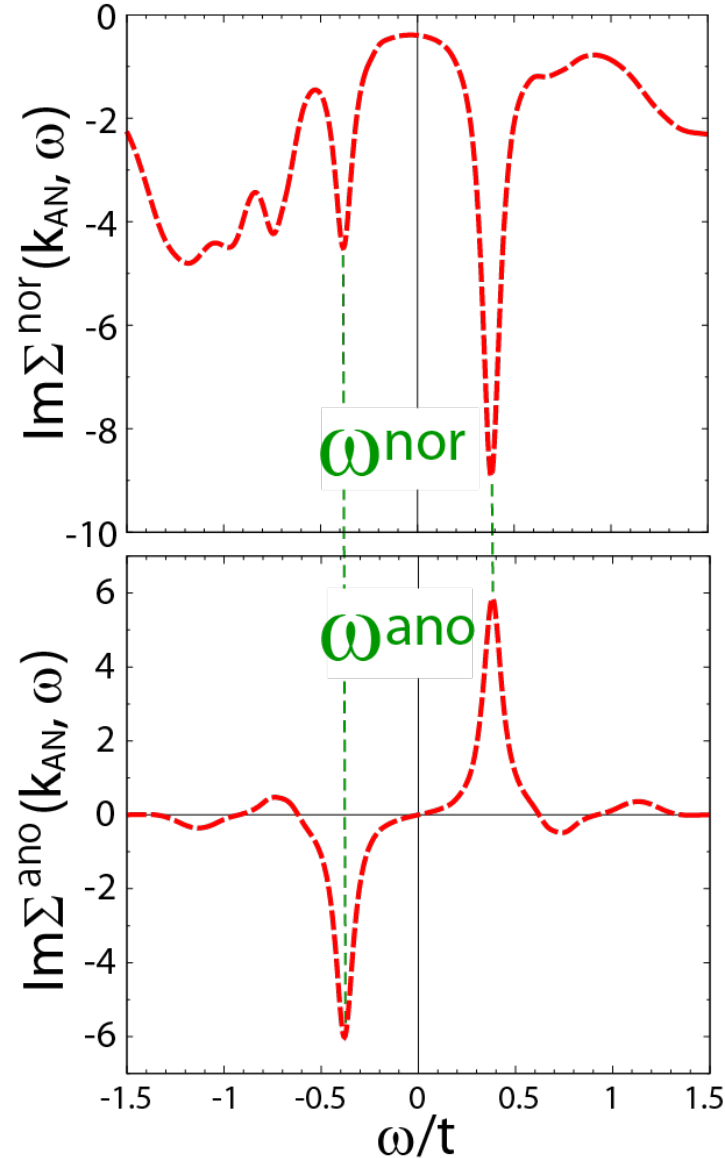
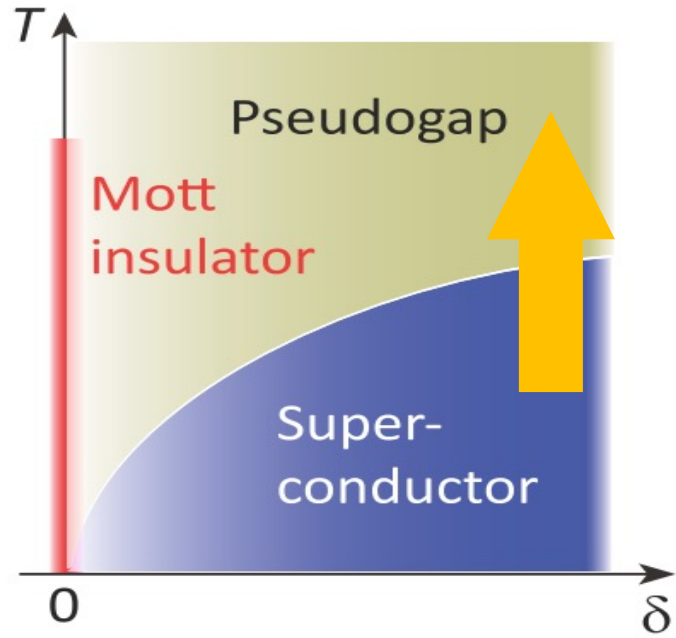
The peak of $\text{Im}\Sigma^{\text{ano}}$ enhances $\text{Re}\Sigma^{\text{ano}}(\omega=0)$ (\sim gap) by 5-10 times.

→ Origin of high T_c

[Maier *et al.*, PRL **100**, 237001 (2008)]

Poles of SC and PG are continuously connected!

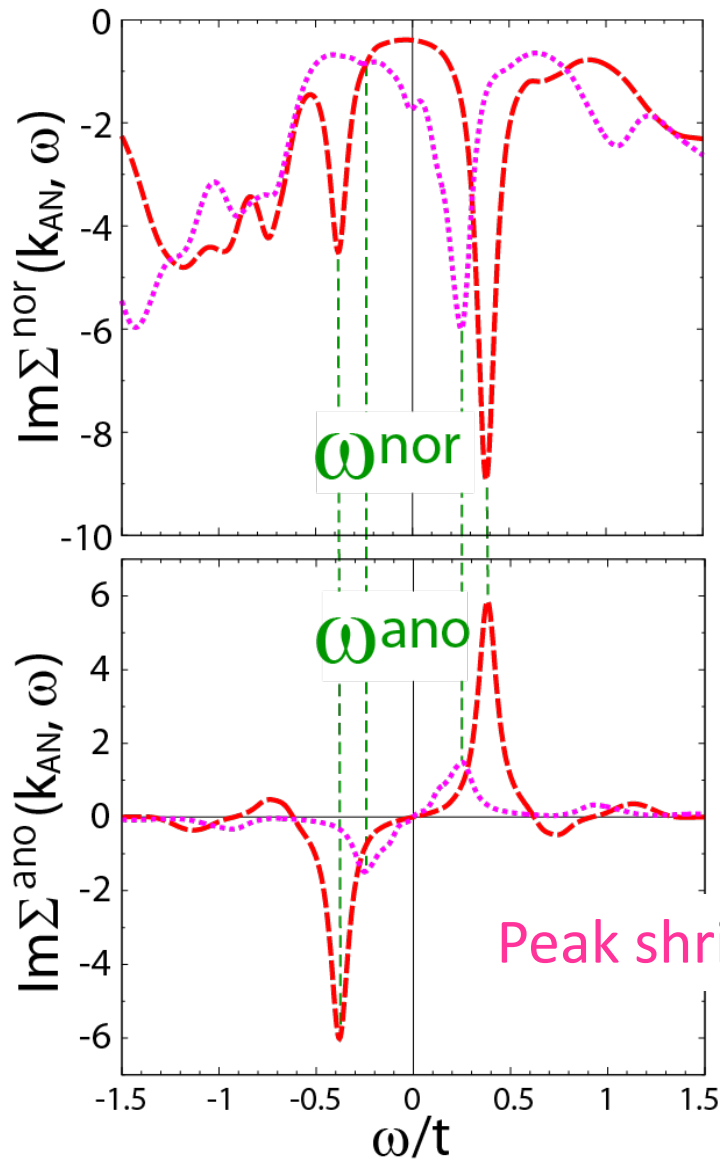
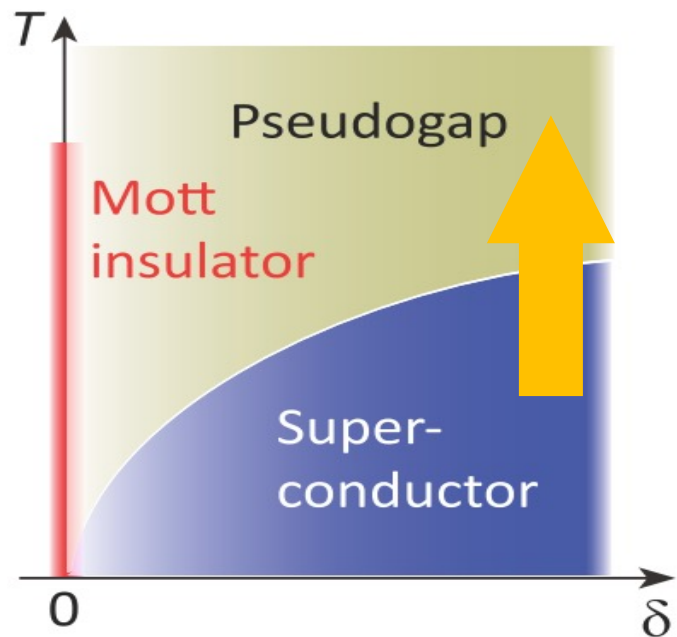
SS, M. Civelli and M. Imada, PRL **116**, 057003 (2016)



$T=0.01$ --- SC

Poles of SC and PG are continuously connected!

SS, M. Civelli and M. Imada, PRL **116**, 057003 (2016)



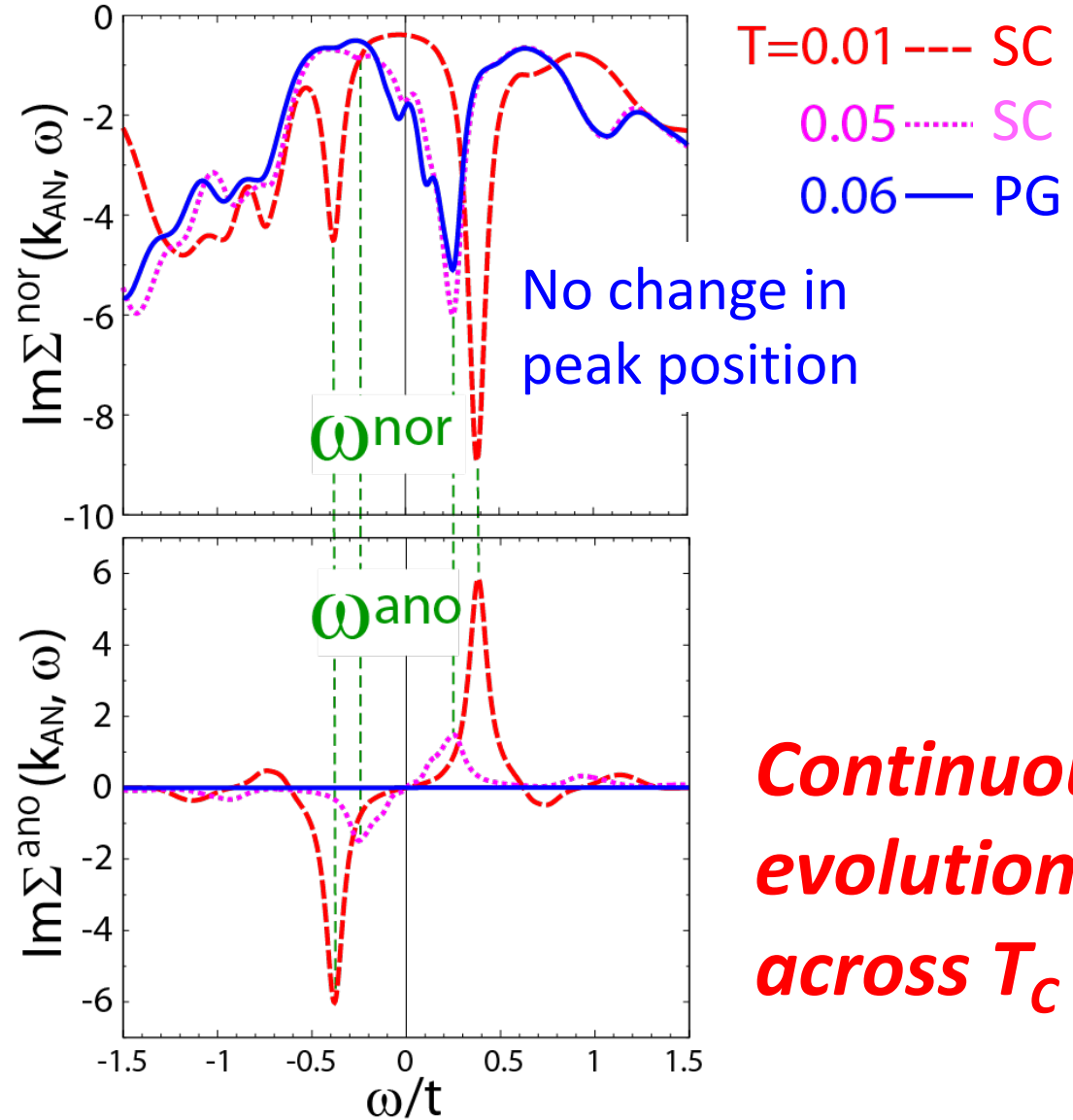
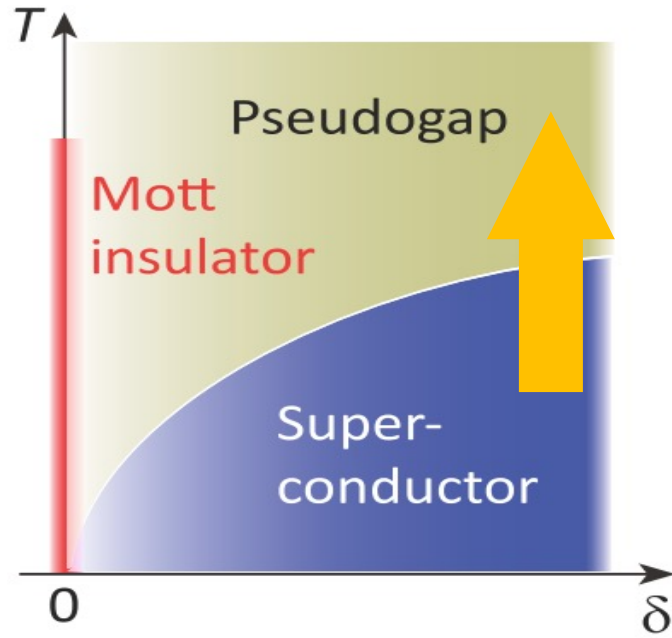
$T=0.01$ --- SC

0.05 SC

Peak shrinks

Poles of SC and PG are continuously connected!

SS, M. Civelli and M. Imada, PRL **116**, 057003 (2016)

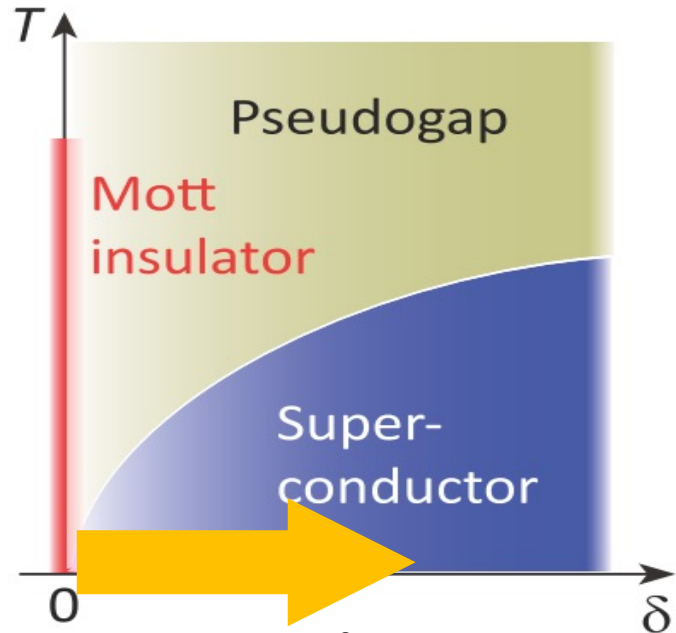


Continuous evolution across T_C

Poles of SC and MI are continuously connected!

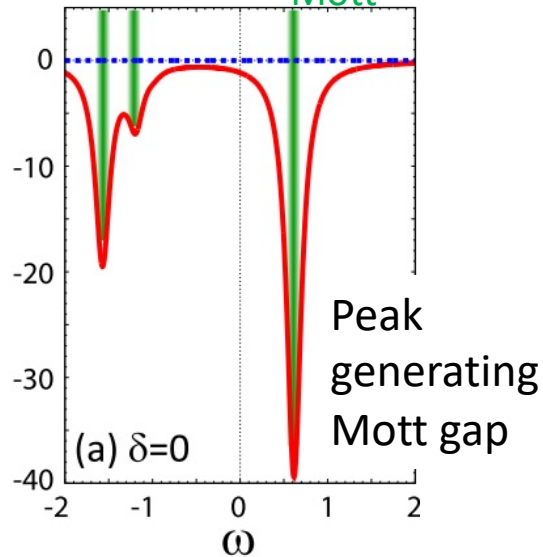
SS, M. Civelli and M. Imada, PRB **98**, 195109 (2018)

Continuous evolution with doping
 - Peak enhancing SC emerges at ω_{Mott} , which characterizes the Mott gap.

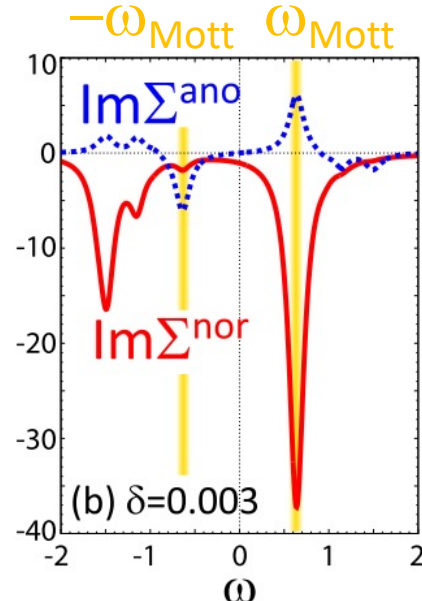


Mott ins.

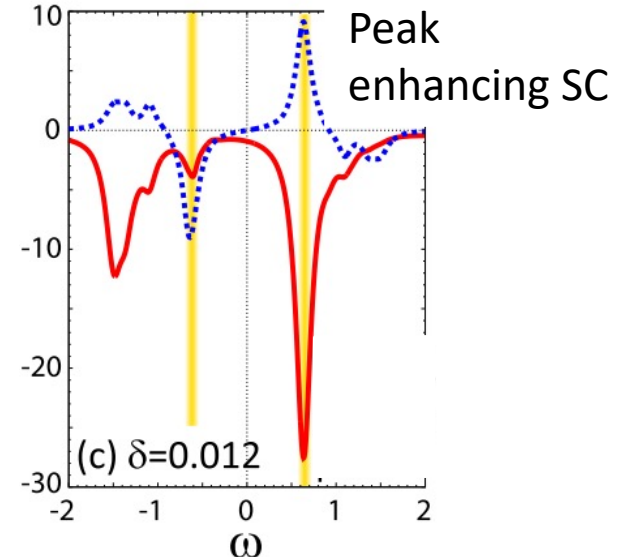
ω_{Mott}

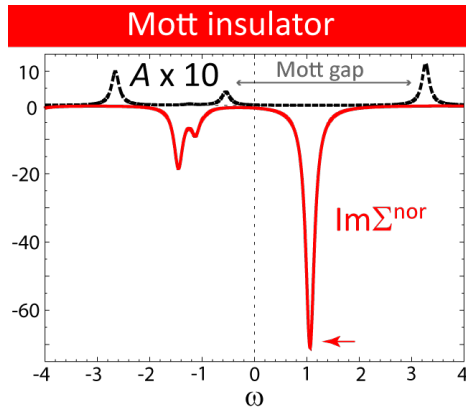


SC

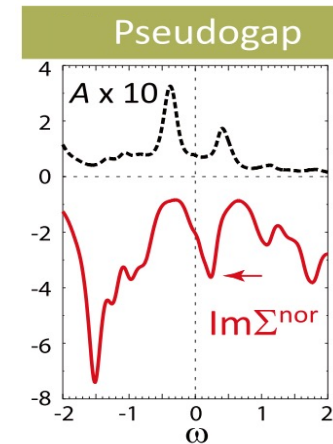
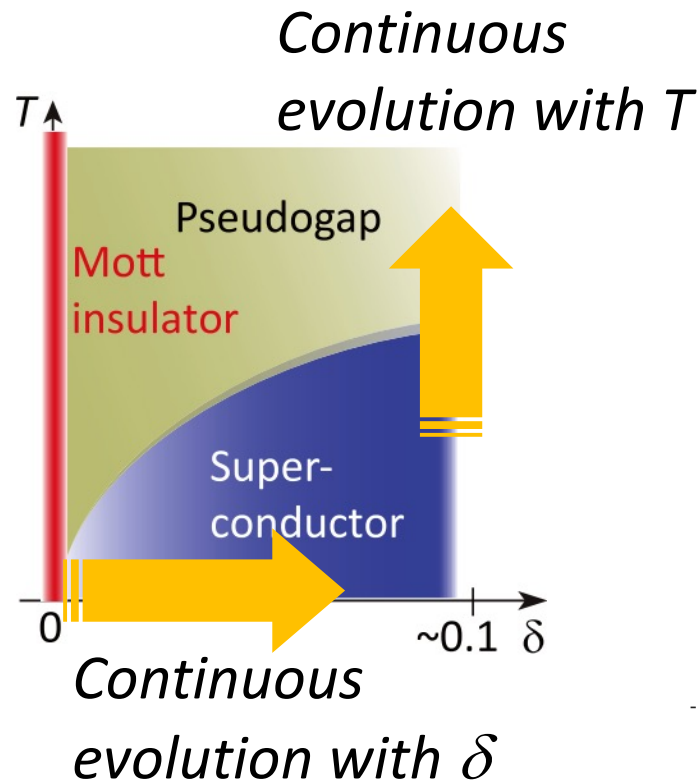


SC

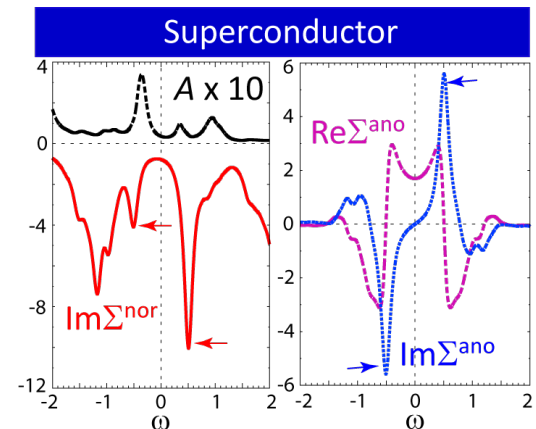




Generating Mott gap



Generating pseudogap



Enhancing T_c

These poles are essentially the same!

- ***Mott physics yields high- T_c SC and pseudogap.***
- ***Relation between high- T_c SC and pseudogap.***

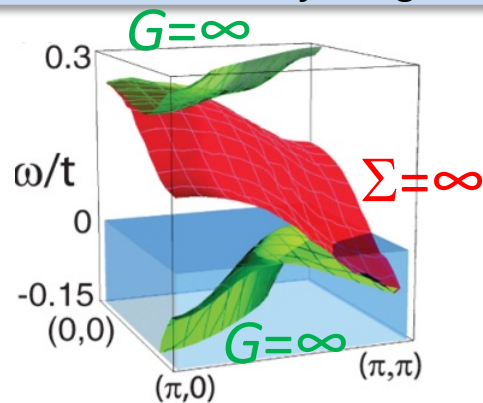
What does the self-energy pole mean?

Self-energy pole = Hidden Fermion

In Mott-insulating or pseudogap state,

$$\Sigma^{\text{nor}}(\mathbf{k}, \omega) \simeq \frac{V(\mathbf{k})^2}{\omega - \varepsilon_f(\mathbf{k})} \quad \omega \rightarrow \omega + i\eta \quad (1)$$

cf. Yang, Rice and Zhang, PRB 73, 174501 (2006)



CDMFT result for Hubbard model
[PRL102, 056404 (2009)]

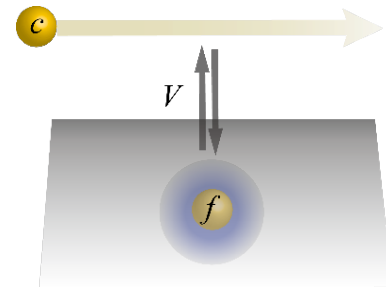
Phenomenological model:

$$H = \sum_{\mathbf{k}\sigma} \left[\varepsilon_c(\mathbf{k}) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \varepsilon_f(\mathbf{k}) f_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma} + V (c_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma} + f_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}) \right]$$

c : Bare electron

f : Hidden fermion (emergent from strong correlation)

Integrating out $f \rightarrow$ Eq. (1)

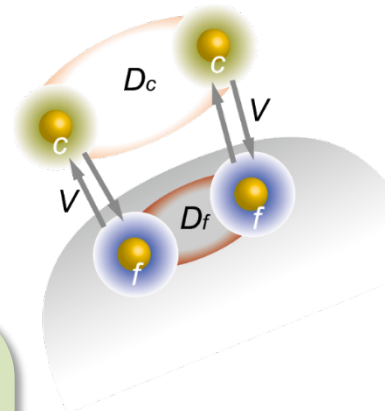


Self-energy pole = Hidden Fermion

Extension of the model to SC state:

$$H = \sum_{\mathbf{k}\sigma} \left[\varepsilon_c(\mathbf{k}) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \varepsilon_f(\mathbf{k}) f_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma} + V (c_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma} + f_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}) \right] \\ - \sum_{\mathbf{k}} \left[D_c(\mathbf{k}) c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} + D_f(\mathbf{k}) f_{\mathbf{k}\uparrow} f_{-\mathbf{k}\downarrow} + \text{h.c.} \right]$$

Integrating out f $\frac{1}{Z^{\text{eff}}} \exp(-S^{\text{eff}}[c^\dagger, c]) = \frac{1}{Z} \int \mathcal{D}f^\dagger \mathcal{D}f \exp(-S[c^\dagger, c, f^\dagger, f])$



$$\Sigma^{\text{nor}}(\mathbf{k}, \omega) = \frac{V(\mathbf{k})^2 (\omega + \varepsilon_f(\mathbf{k}))}{\omega^2 - \varepsilon_f(\mathbf{k})^2 - D_f(\mathbf{k})^2}$$

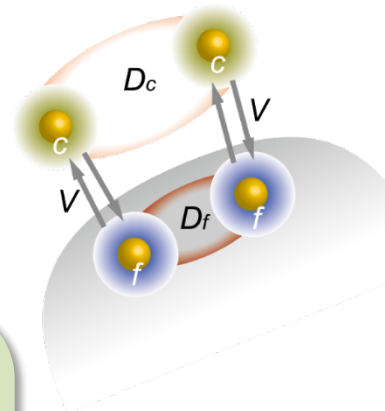
$$\Sigma^{\text{ano}}(\mathbf{k}, \omega) = D_c(\mathbf{k}) - \frac{V(\mathbf{k})^2 D_f(\mathbf{k})}{\omega^2 - \varepsilon_f(\mathbf{k})^2 - D_f(\mathbf{k})^2}$$

Self-energy pole = Hidden Fermion

Extension of the model to SC state:

$$H = \sum_{\mathbf{k}\sigma} \left[\varepsilon_c(\mathbf{k}) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \varepsilon_f(\mathbf{k}) f_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma} + V (c_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma} + f_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}) \right] \\ - \sum_{\mathbf{k}} \left[D_c(\mathbf{k}) c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} + D_f(\mathbf{k}) f_{\mathbf{k}\uparrow} f_{-\mathbf{k}\downarrow} + \text{h.c.} \right]$$

Integrating out f $\frac{1}{Z^{\text{eff}}} \exp(-S^{\text{eff}}[c^\dagger, c]) = \frac{1}{Z} \int \mathcal{D}f^\dagger \mathcal{D}f \exp(-S[c^\dagger, c, f^\dagger, f])$



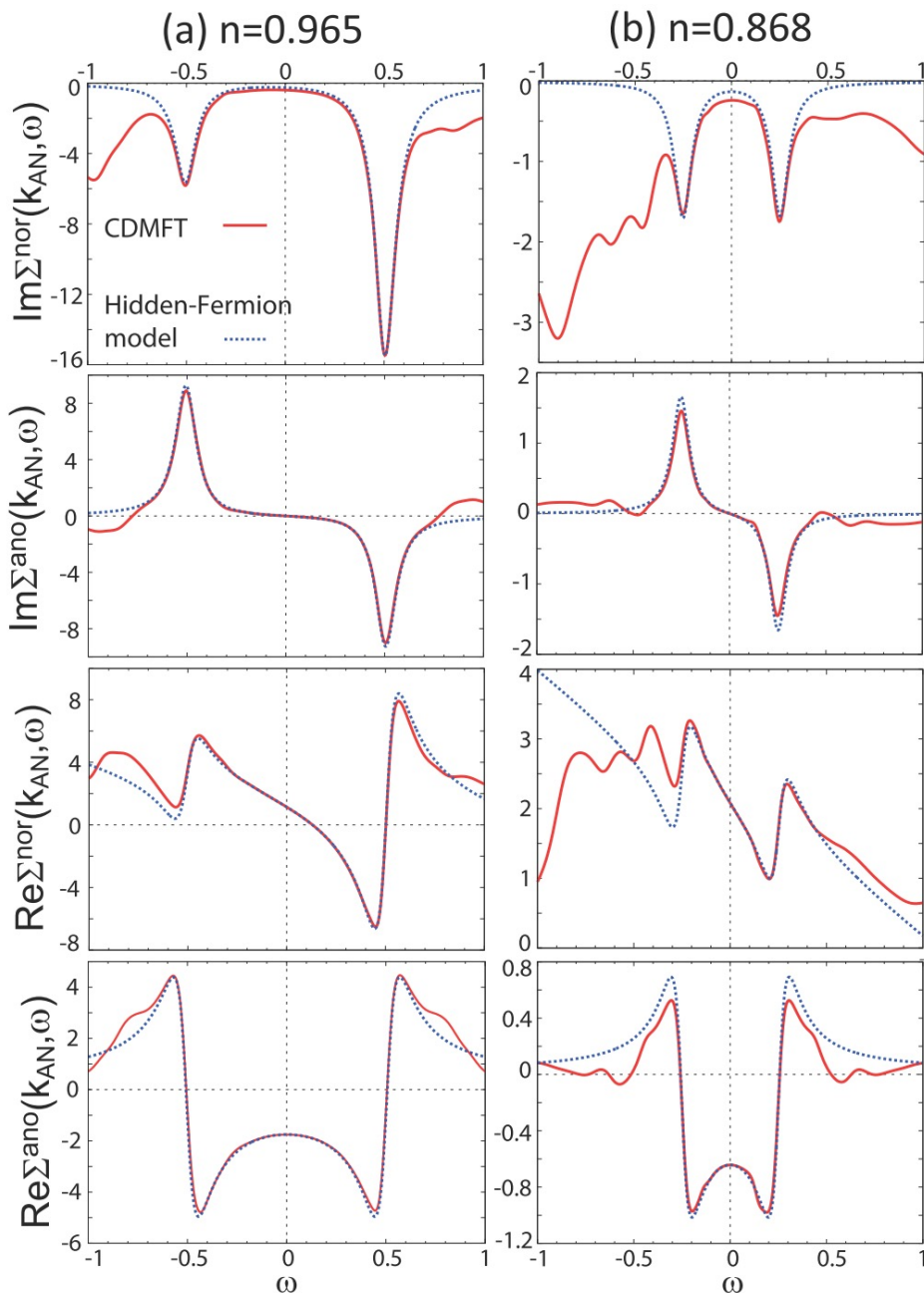
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$$\Sigma^{\text{ano}}(\mathbf{k}, \omega) = D_c(\mathbf{k}) - \frac{V(\mathbf{k})^2 D_f(\mathbf{k})}{\omega^2 - \varepsilon_f(\mathbf{k})^2 - D_f(\mathbf{k})^2} \quad \leftarrow \text{Poles due to } D_f$$

Poles at the same ω 's, in consistency with CDMFT.

Fitting of low-energy part of self-energy

$U=8t$
 $t'=-0.2t$
 $T=0.01$
 $\mathbf{k}=\mathbf{k}_{AN}=(\pi,0)$



SS, M. Civelli and M. Imada,
PRB **94**, 115130 (2016)

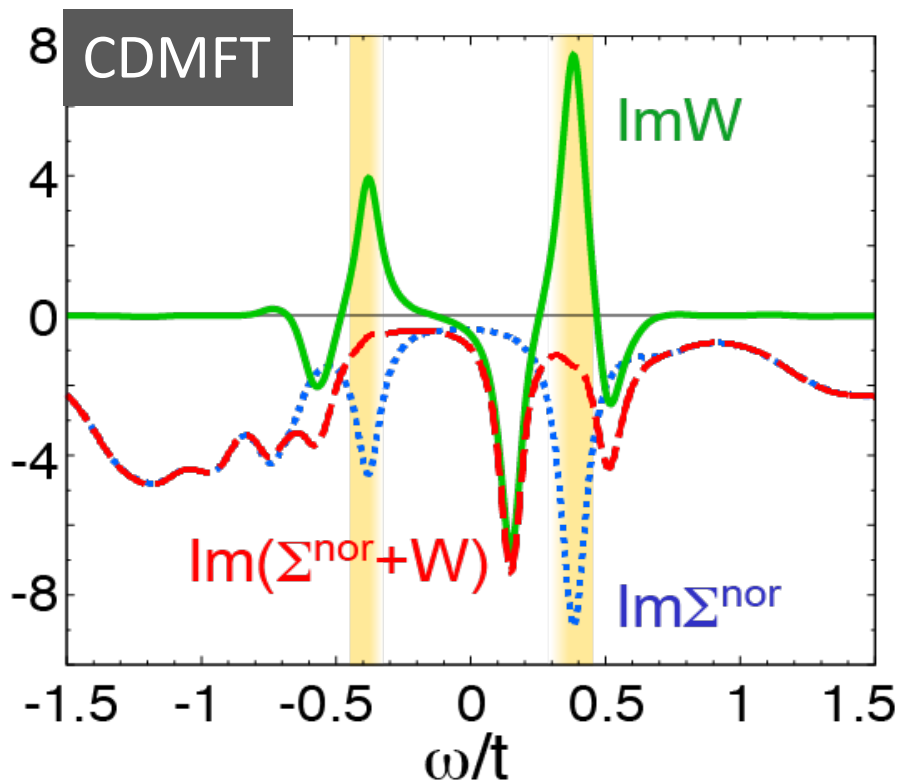
*Low-energy part
is well fitted by
hidden-fermion
model.*

Pole-to-pole cancellation in G

SS, M. Civelli and M. Imada, PRL **116**, 057003 (2016)

$$G(\mathbf{k}, \omega) = \left[\omega + \mu - \varepsilon_{\mathbf{k}} - \Sigma^{\text{nor}}(\mathbf{k}, \omega) - W(\mathbf{k}, \omega) \right]^{-1}$$

$$W(\mathbf{k}, \omega) = \frac{\Sigma^{\text{ano}}(\mathbf{k}, \omega)^2}{\omega - \mu + \varepsilon_{\mathbf{k}} + \Sigma^{\text{nor}}(\mathbf{k}, -\omega)^*}$$



Σ^{nor} cancels with W

Residues at the poles
in hidden-fermion model

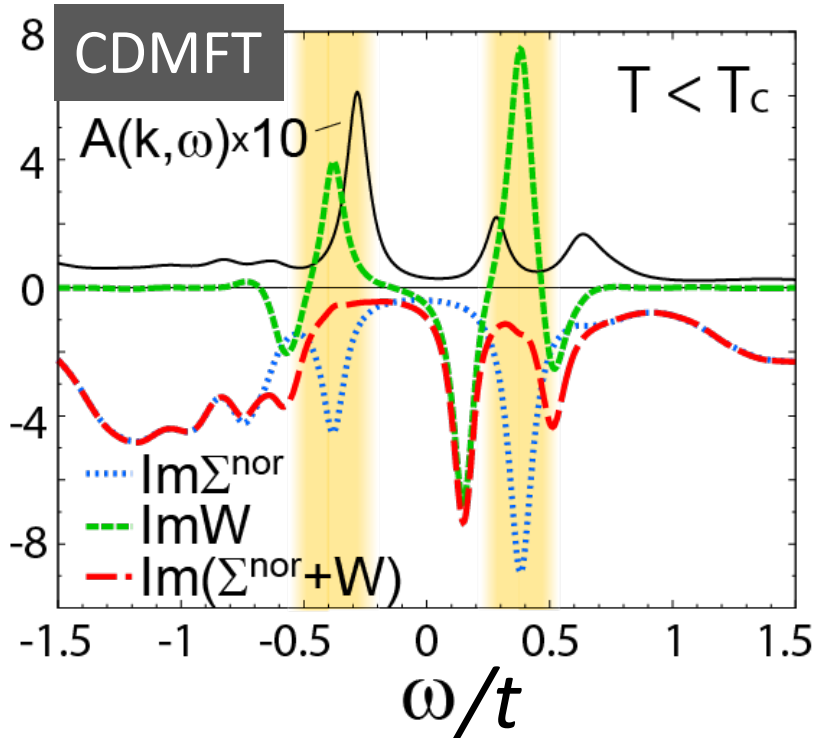
$$\begin{aligned} \text{Res}_{\Sigma^{\text{nor}}} &= \frac{V^2}{2} \left(1 \pm \frac{\varepsilon_f}{\sqrt{\varepsilon_f^2 + D_f^2}} \right) \\ &= -\text{Res}_W \end{aligned}$$

Fully consistent!

What do these results mean?

Pole-to-pole cancellation

SS, M. Civelli and M. Imada, PRL **116**, 057003 (2016)



Σ^{nor} cancels with W



Self-energy peaks are invisible in G (and hence in spectra).

This explains why the self-energy peak has eluded an experimental detection.

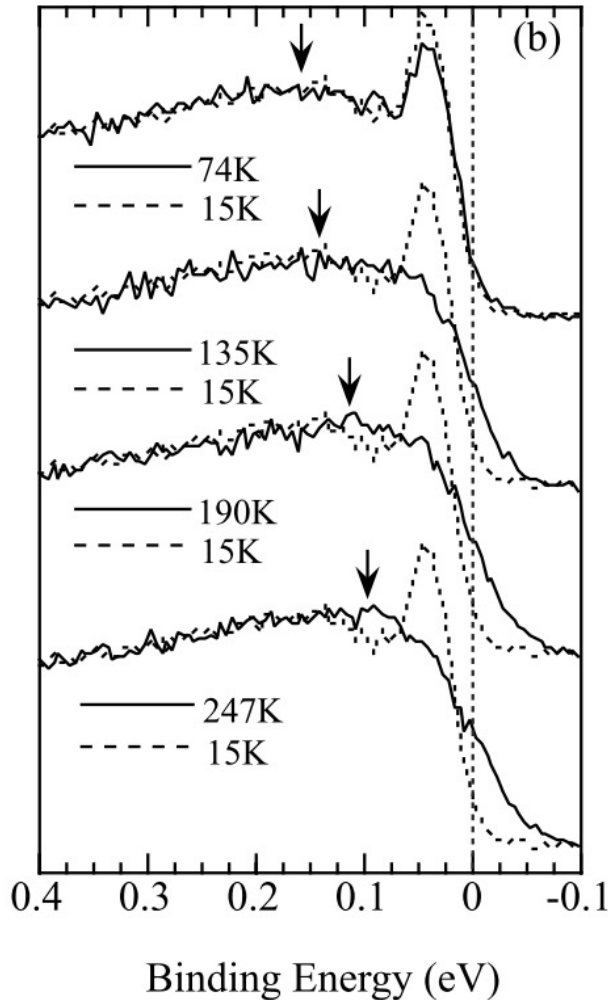
Recently detected by ARPES+Machine learning! [Y. Yamaji *et al.*, arXiv: 1903.08060]

Bogoliubov peak can emerge from broad spectra

How can a coherent Bogoliubov peak emerge from a broad spectrum lacking quasiparticles?

➔ Cancellation, i.e.,

- $T > T_c$
Large $\text{Im}\Sigma^{\text{nor}}$ → Pseudogap & broad spectra
- $T < T_c$
 Σ^{nor} is **anceled** with W .
→ Sharp Bogoliubov peak.

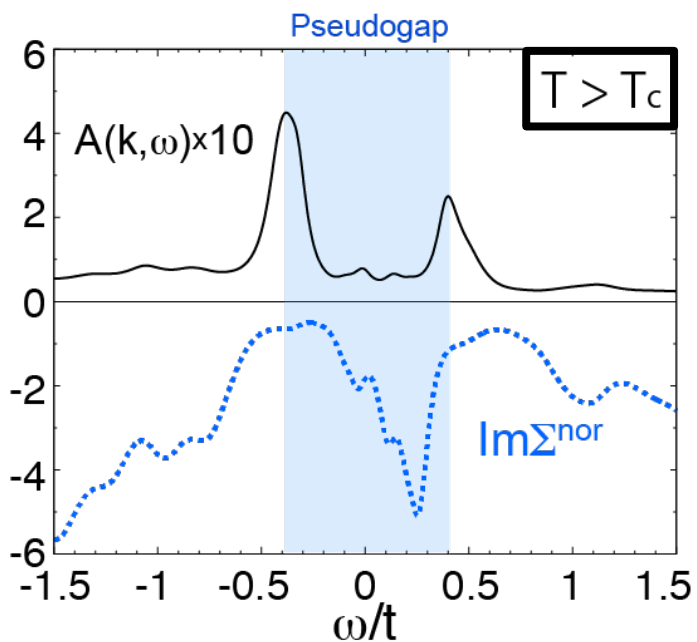
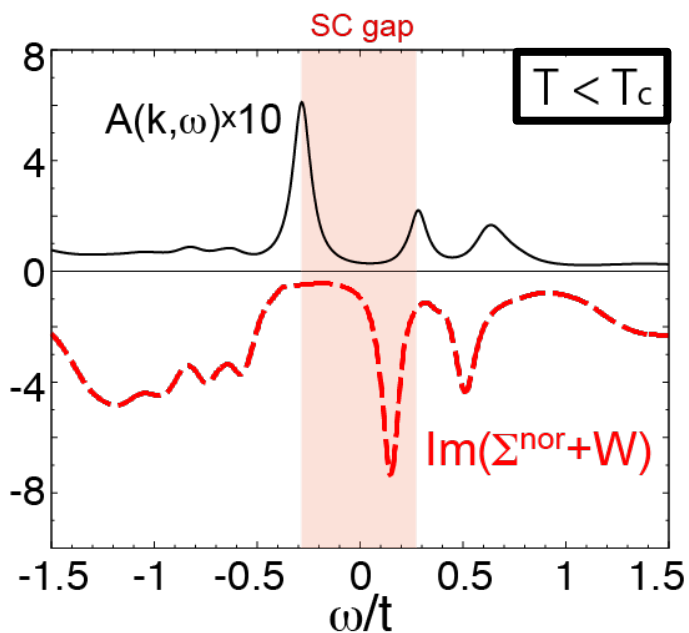


ARPES for Bi2212, UD89K, $\mathbf{k}=(\pi,0)$
Campuzano *et al.*, PRL **83**, 3709 (1999)

Another consequence: Peak-dip-hump
[PRL **116**, 057003; PRL **116**, 197001 (2016)]

PG and SC gap involve different singularities

i.e., mathematically different!



$$G(\mathbf{k}, \omega) = \left[\omega + \mu - \varepsilon_{\mathbf{k}} - \Sigma^{\text{nor}}(\mathbf{k}, \omega) - W(\mathbf{k}, \omega) \right]^{-1}$$

$$W(\mathbf{k}, \omega) = \frac{\Sigma^{\text{ano}}(\mathbf{k}, \omega)^2}{\omega - \mu + \varepsilon_{\mathbf{k}} + \Sigma^{\text{nor}}(\mathbf{k}, -\omega)^*}$$

SC state

Gap ($G=0$) \leftrightarrow $\left\{ \begin{array}{l} \Sigma^{\text{nor}}(\mathbf{k}, \omega) = \infty \\ \text{or} \\ \Sigma^{\text{ano}}(\mathbf{k}, \omega) = \infty \end{array} \right\}$ *Cancelling!*

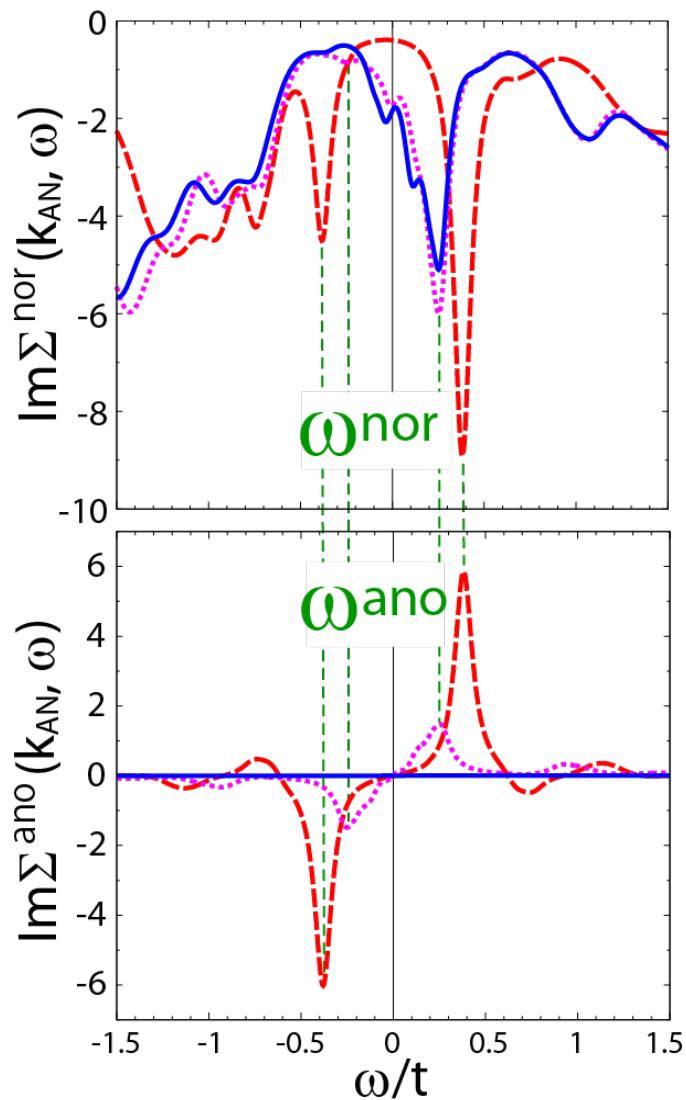
$\omega - \mu + \varepsilon_{\mathbf{k}} + \Sigma^{\text{nor}}(\mathbf{k}, -\omega)^* = 0$ **SC gap**

Normal state

$\Sigma^{\text{ano}} = W = 0$: No cancellation any more.

Gap ($G=0$) \leftrightarrow $\Sigma^{\text{nor}}(\mathbf{k}, \omega) = \infty$ **Pseudogap**

However, the same hidden fermion is at the origin of both pseudogap and 'high T_c ':



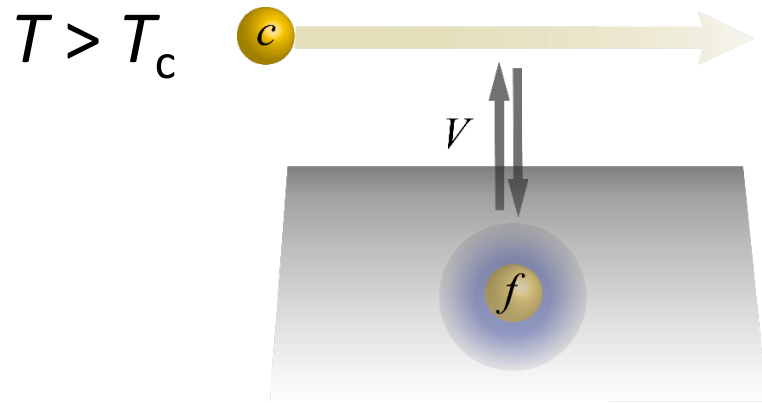
$T=0.01$ --- SC
 0.05 SC
 0.06 — Pseudogap

Peak enhancing SC

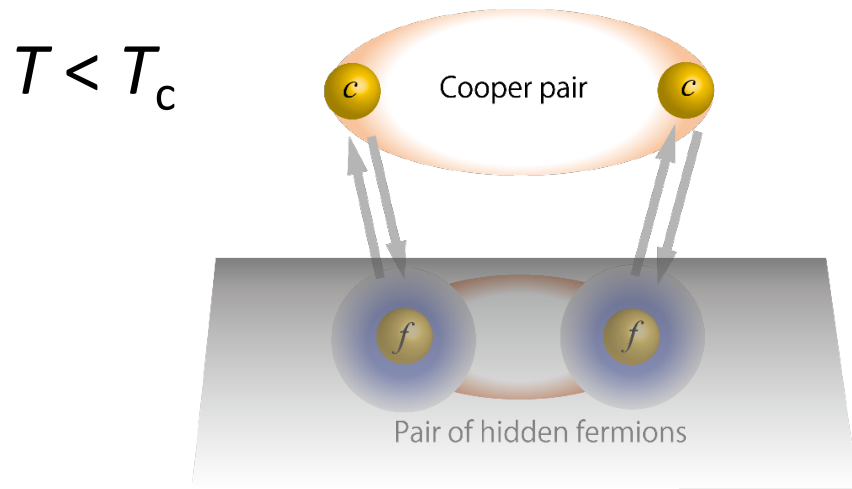
Continuous evolution with T

Peak generating PG

Unified understanding of pseudogap and high- T_c superconductivity



Pseudogap
= (Hybridization gap with f)



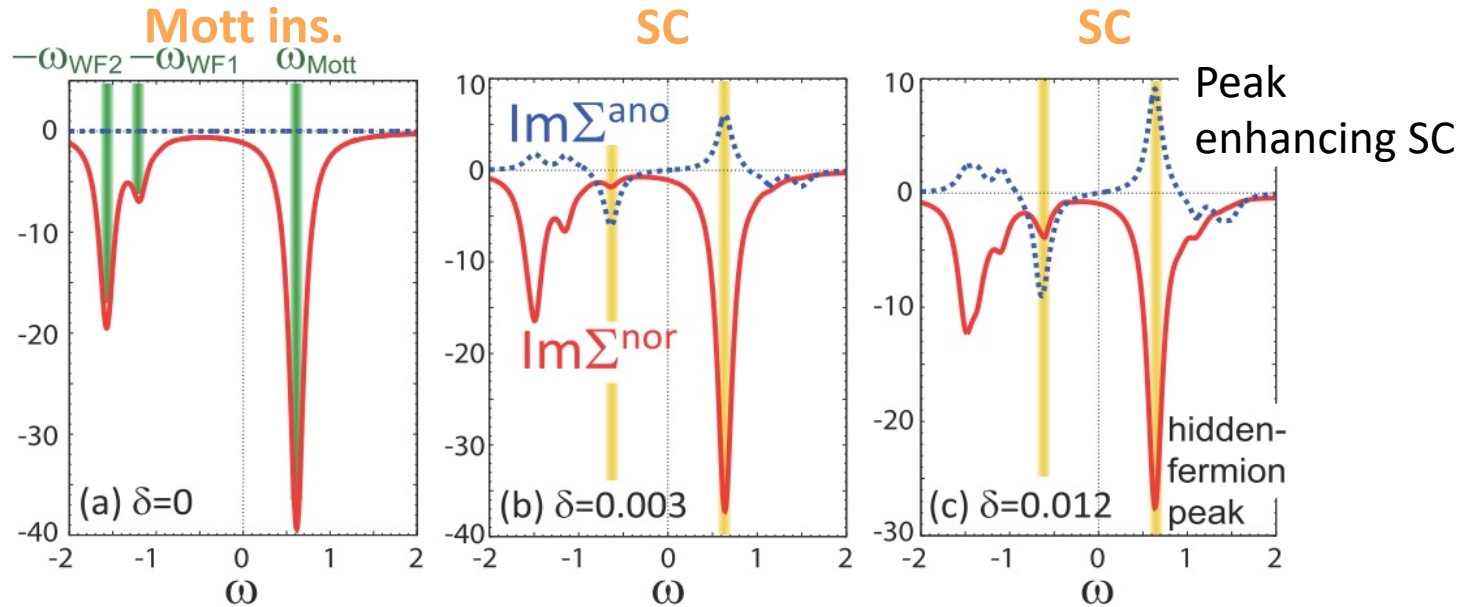
High- T_c SC
Hybridization with f
 \rightarrow Pole of Σ^{ano} \rightarrow High T_c

Fermionic high- T_c mechanism



Bosonic glue ("conventional")

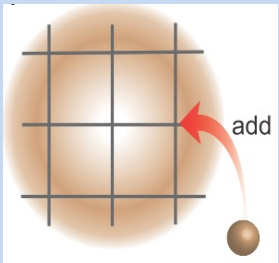
Origin of the hidden fermion f is in the Mott gap



Direct microscopic relation between MI and high- T_c SC.

Two different low-energy excitations in doped Mott insulators

[Yamaji and Imada, PRL **106**, 016404 (2011)]



Electron

Extended in space \rightarrow Quasiparticle

or

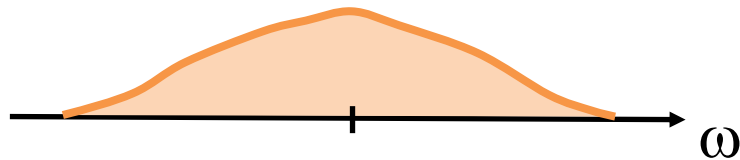
Weakly bound to the hole \rightarrow **Hidden fermion**

Conventional superconductor

BCS theory tells a relation in the spectral function

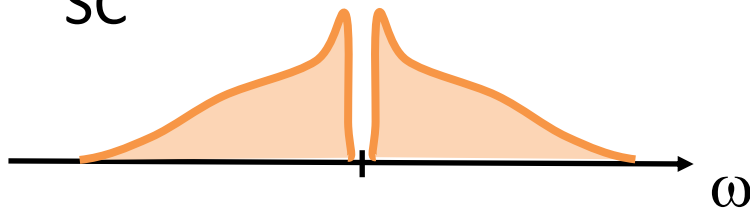
DOS

Metal



$$T_c \propto \exp\left[-\frac{1}{VD(0)}\right]$$

SC

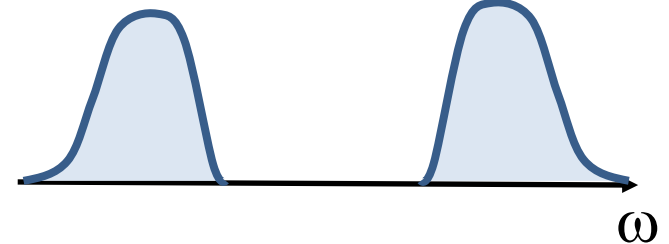


Strongly-correlated superconductor

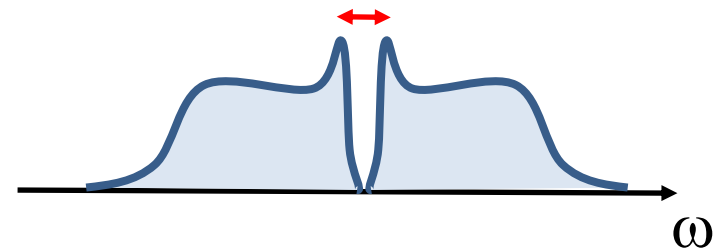
Direct connection in spectra is hard to imagine.

DOS

Mott gap $\sim U$



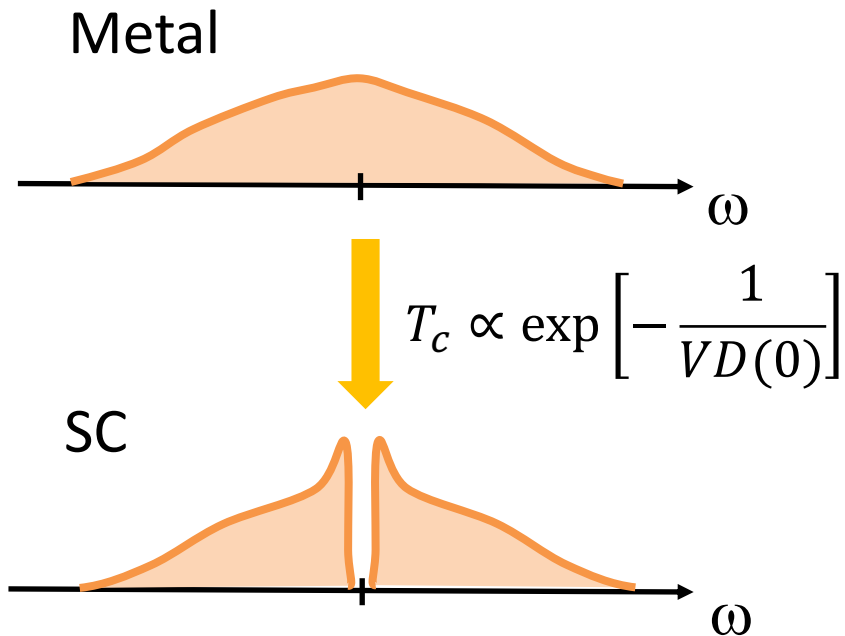
Superconducting gap $\ll U$



Conventional superconductor

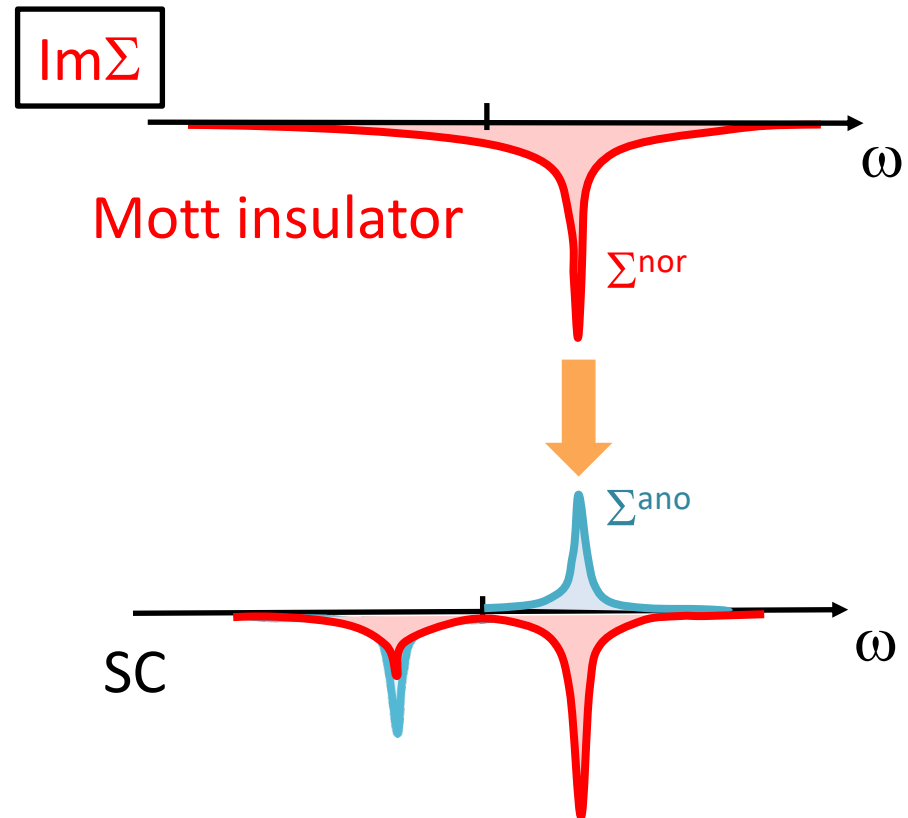
BCS theory tells a relation in the spectral function

DOS



Strongly-correlated superconductor

We found a relation in the **self-energy**

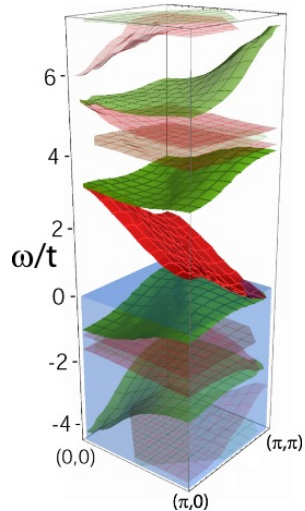


Property of SC determined by *insulator*

Summary

Low-energy dynamics in MI, PG and SC states is governed by a self-energy pole (= hidden fermion)!

Pole of Σ^{nor} generates Mott gap

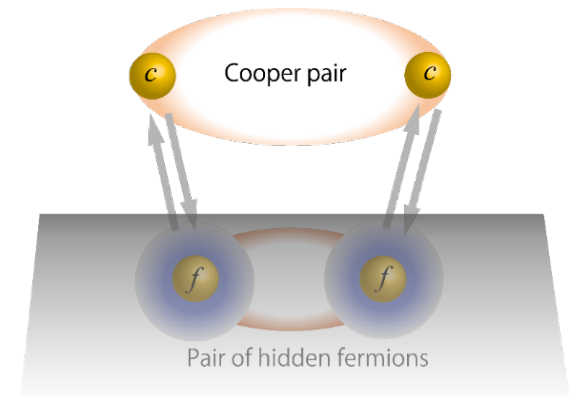
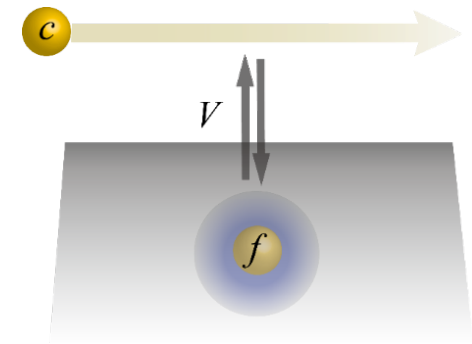


Pole of Σ^{nor} generates PG



Σ^{ano} vanishes above T_c

Pole of Σ^{ano} enhances SC



PRL 116, 057003 (2016)
PRB 94, 115130 (2016)
PRB 98, 195109 (2018)