Superconductivity in Quasicrystals

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Discovery of superconductivity in quasicrystal

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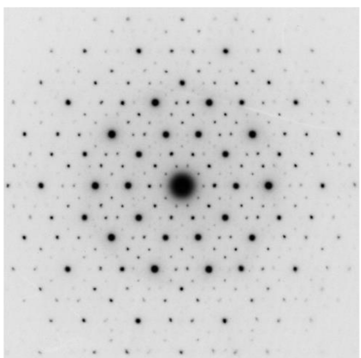
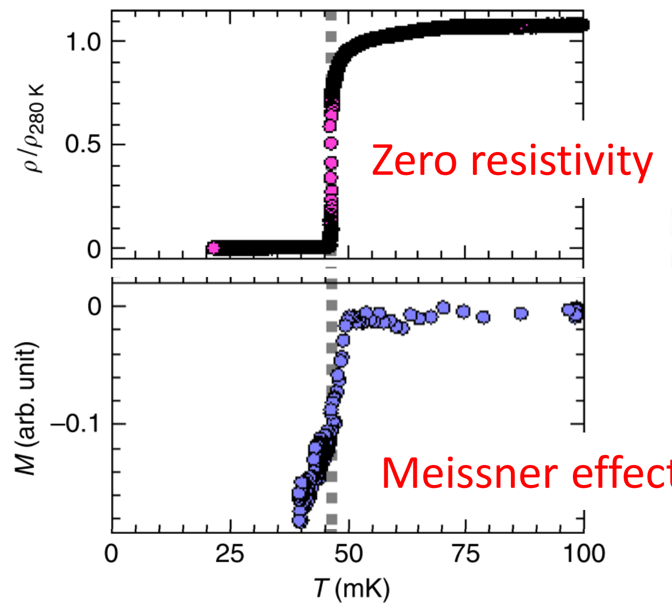
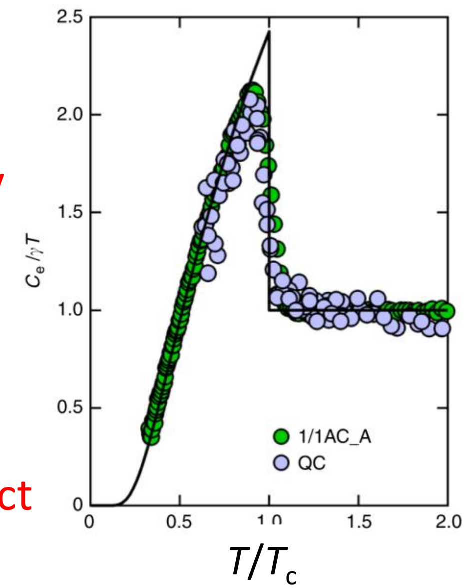
Al-Zn-Mg alloy

T_c=50mK



Structure identification





Bulk property

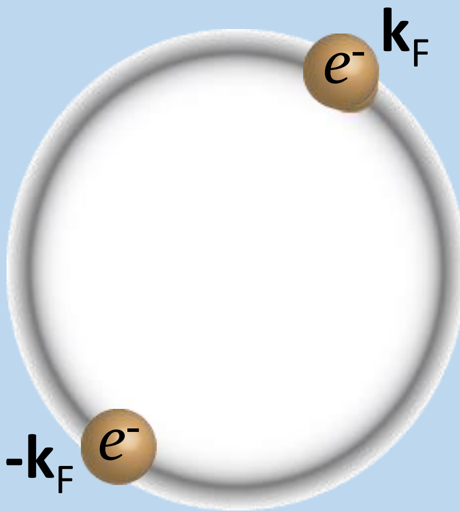
First example of electronic long-range order in QC

Top 10 Breakthroughs of 2018 in Physics World

What’s the issue?

Standard understanding of superconductivity

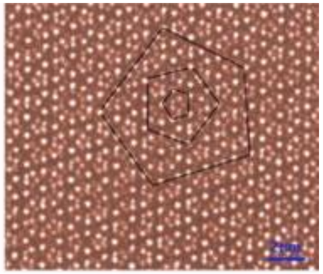
- Presence of Fermi surface
- Cooper pair
  = (2 el. with $\mathbf{k}_F$ and $-\mathbf{k}_F$)
- Many properties calculated
  in momentum space.



Quasicrystal: No momentum space, no Fermi surface

How can we understand a superconducting QC?

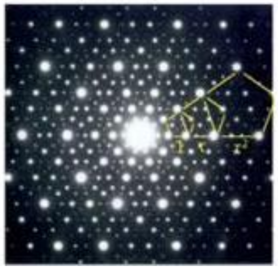
Note: Two different reciprocal spaces



$$\rho(\boldsymbol{r}) = \left\langle c_{\boldsymbol{r}}^{\dagger} c_{\boldsymbol{r}} \right\rangle$$

Local density

Figures from https://www.kek.jp/ja/newsroom/2011/12/08/1200/



$$S(\boldsymbol{q}) = \int \rho(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}} d\boldsymbol{r}$$

q: F. T. of *absolute* coordinate **r**

In periodic systems, Fermi surface is defined by a peak in

$$A(\boldsymbol{k}, \omega = E_F) = -\frac{1}{\pi} \mathrm{Im} G(\boldsymbol{k}, \omega = E_F)$$

$$\mathrm{F.T.\,of}\; G(\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2, t) = -i\left\langle T c_{\boldsymbol{r}_1}(t) c_{\boldsymbol{r}_2}^{\dagger}(0) \right\rangle$$

k: F. T. of *relative* coordinate

This is not well defined in QC.

What's the issue?

Anderson's theorem

P. Anderson, J. Phys. Chem. Solids **11**, 26 (1959)

s-wave superconductivity is robust against weak (nonmagnetic) disorder

But, strong disorder can destroy SC!

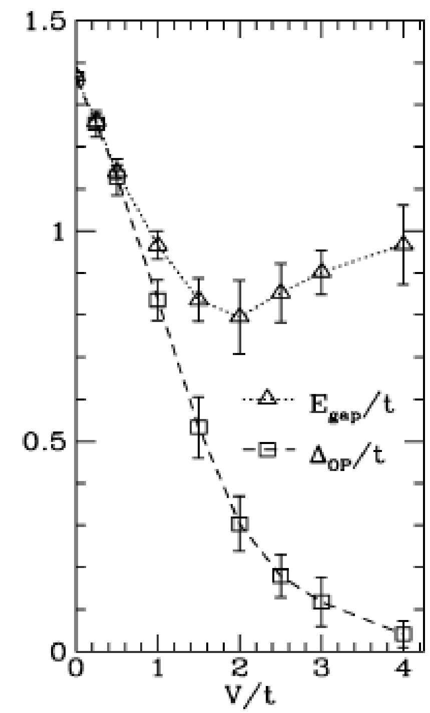


Figure from A. Ghosal, M. Randeria, and N. Trivedi, PRL **81**, 3940 (1998).

Normal state: metal → Anderson insulator

What about quasicrystal?

Normal state: critical wave function

What's the issue?

Quasicrystal

Self-similarity (fractal)

Superconductivity

Macroscopic quantum state

Novel SC properties?

Fractal superconductivity!

How to address the issue?

- No momentum space
- Nonuniform (but not random)

DFT for approximants?

M. Saito, T. Sekikawa, and Y. Ono,
Phys. Status Solidi B 2000108 (2020):
Conductivity and specific heat

Simplified model for QC?

Essence:
- Quasiperiodicity
- Pairing attraction

Our approach

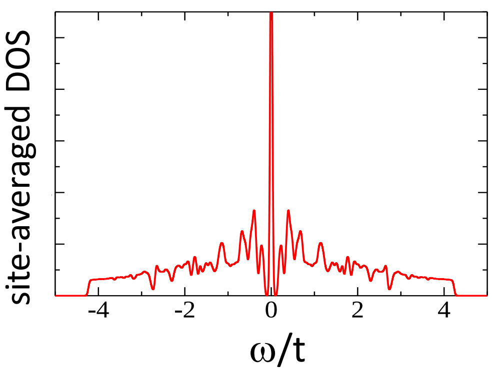
cf. 1D: M. Tezuka and A. M. Garcia-Garcia, PRA **82**, 043613 (2010).

Model of quasiperiodicity

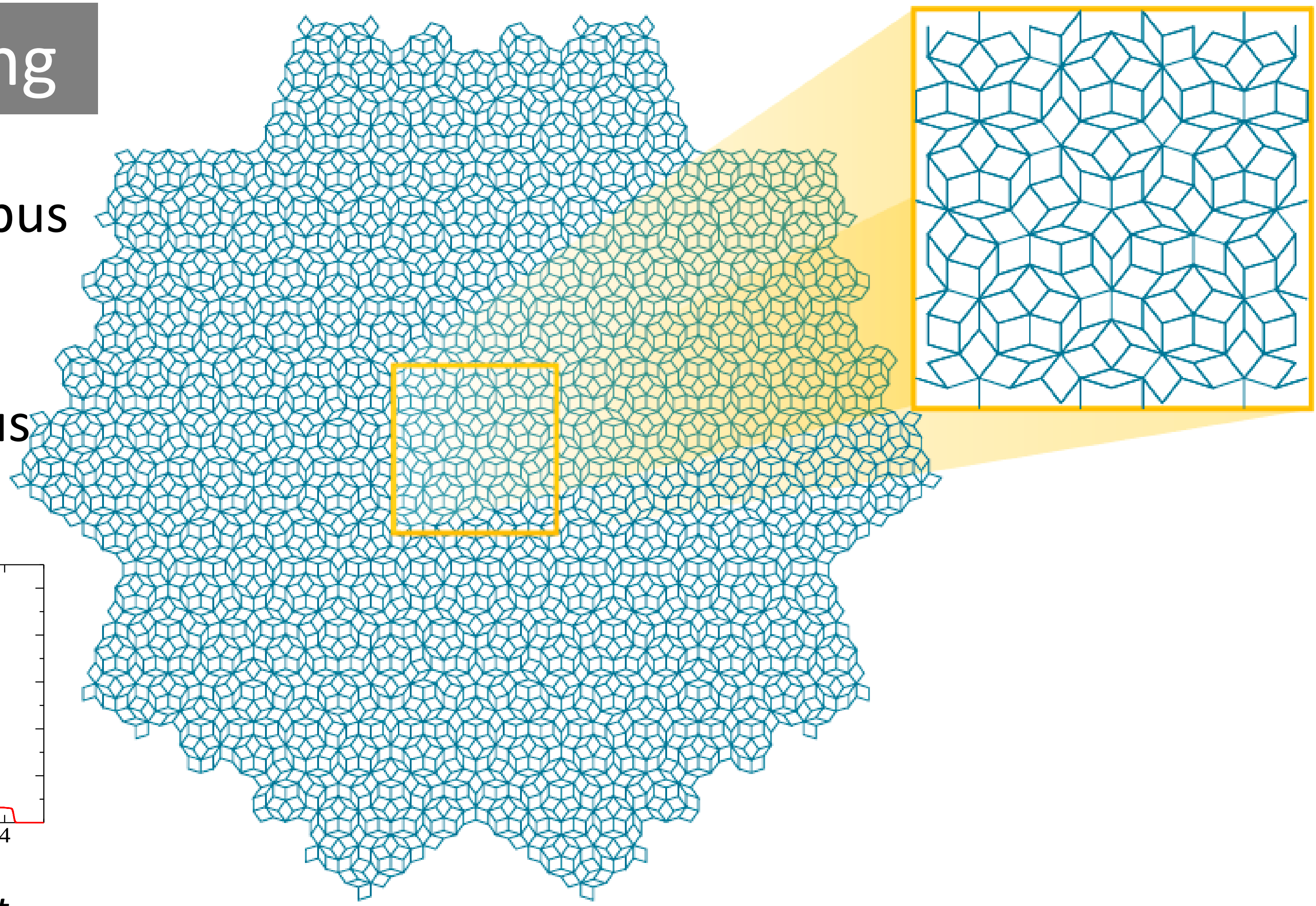
Penrose tiling

Vertex of rhombus
→ lattice point

Edge of rhombus
→ hopping *t*



“Bandwidth” = 8.46*t*



Five-fold rotational symmetry

R. Penrose, Inst. Math. Appl. Bull. **10**, 266 (1974)

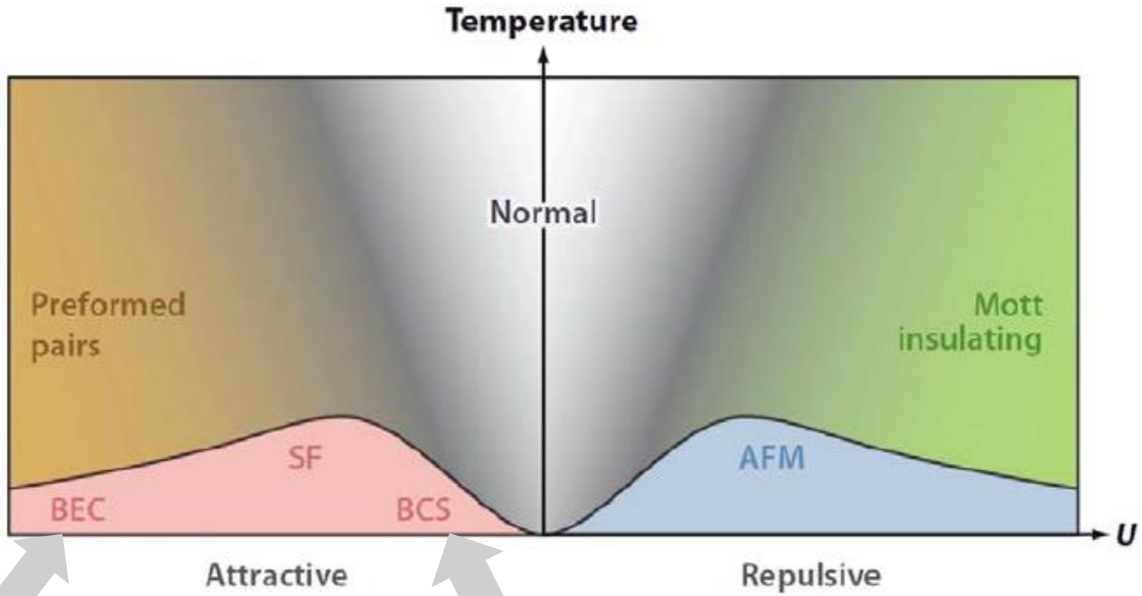
Model with pairing attraction

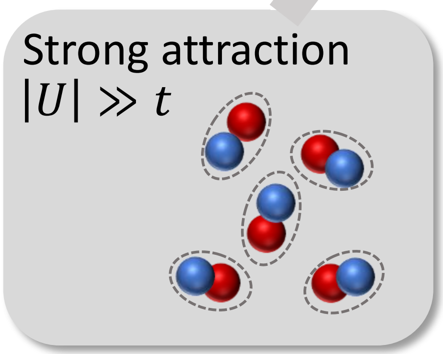
Attractive Hubbard model

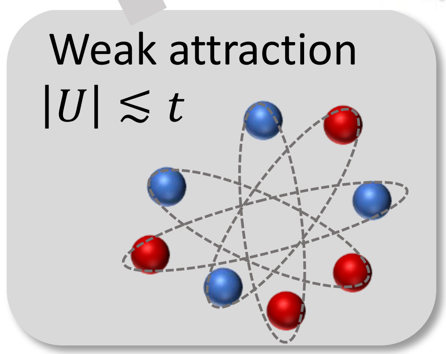
$$H = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} - \mu \sum_{i\sigma} n_{i\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

$U < 0$

Cubic lattice
Half filling







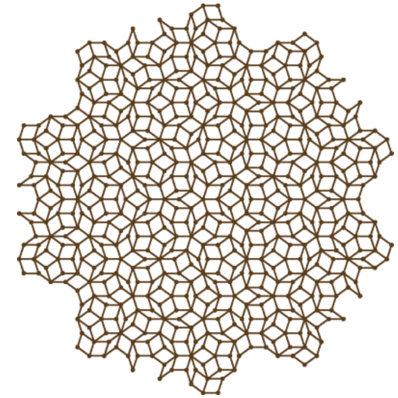
- SC at low T for any U<0.
- BCS-BEC crossover with U.

Figure from Esslinger, Ann. Rev. Condens. Matt. Phys. **1**, 129 (2010)

Attractive Hubbard model on Penrose tiling

$$H = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} - \mu \sum_{i\sigma} n_{i\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

on



Inhomogeneity → Real-space approaches

Bogoliubov-de Gennes theory (BdG)

- Static mean field (one-body approx., weak *U*)
- Large size ~ 1 million sites : Y. Nagai, JPSJ **89**, 074703 (2020)

Real-space dynamical mean-field theory (RDMFT)

- Dynamical mean field (many-body physics, weak-to-strong *U*)
- < 10,000 sites

A. Georges *et al.*, RMP **68**, 13 (1996)
M. Potthoff and W. Nolting, PRB **59**, 2549 (1999)

Bogoliubov - de Gennes theory (BdG)

Eigenenergy
Eigenstates

$$n_{i\sigma} = \left\langle c_{i\sigma}^{\dagger} c_{i\sigma} \right\rangle$$
$$\Delta_i = U \left\langle c_{i\uparrow} c_{i\downarrow} \right\rangle$$

Site-dependent local quantities

$$\hat{H}_{BdG} = \begin{pmatrix} Un_{1\downarrow} - \mu & -t & \cdots & \Delta_1 & 0 & \cdots \\ -t & Un_{2\downarrow} - \mu & \cdots & 0 & \Delta_2 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ \Delta_1 & 0 & \cdots & -Un_{1\uparrow} + \mu & t & \cdots \\ 0 & \Delta_2 & \cdots & t & -Un_{2\uparrow} + \mu & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

: 2N x 2N matrix

Real-space dynamical mean-field theory (RDMFT)

Site-dependent impurity problem

$$\hat{g}_0(i\omega_n)^{-1} = \hat{G}_{ii}(i\omega_n)^{-1} + \hat{\Sigma}_i(i\omega_n)$$

Exact diag.

$$\hat{\Sigma}_i(i\omega_n) = \begin{pmatrix} \Sigma_i^{nor}(i\omega_n) & \Sigma_i^{ano}(i\omega_n) \\ \Sigma_i^{ano}(i\omega_n) & -\Sigma_i^{nor}(-i\omega_n) \end{pmatrix}$$

Site- & energy-dependent local self-energy

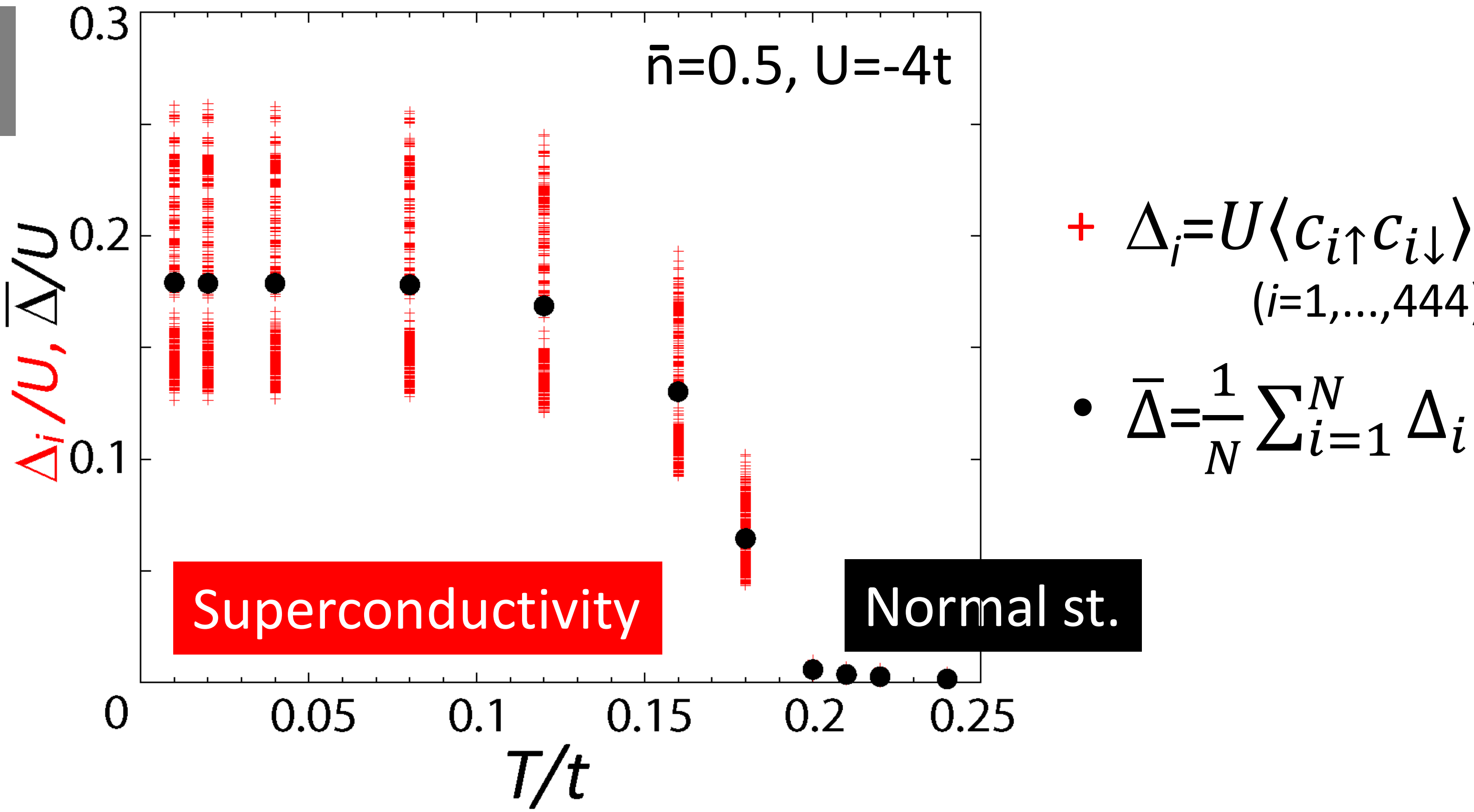
$$\hat{G}(i\omega_n)^{-1} = \begin{pmatrix} i\omega_n + \mu - \Sigma_1^{nor} & -t & \cdots & -\Sigma_1^{ano} & 0 & \cdots \\ -t & i\omega_n + \mu - \Sigma_2^{nor} & \cdots & 0 & -\Sigma_2^{ano} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ -\Sigma_1^{ano} & 0 & \cdots & i\omega_n - \mu + \Sigma_1^{nor} & t & \cdots \\ 0 & -\Sigma_2^{ano} & \cdots & t & i\omega_n - \mu + \Sigma_2^{nor} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

: 2N x 2N matrix

- *Geometry of the Penrose lattice comes in the one-body part.*
- *Nonlocal correlations are neglected.*

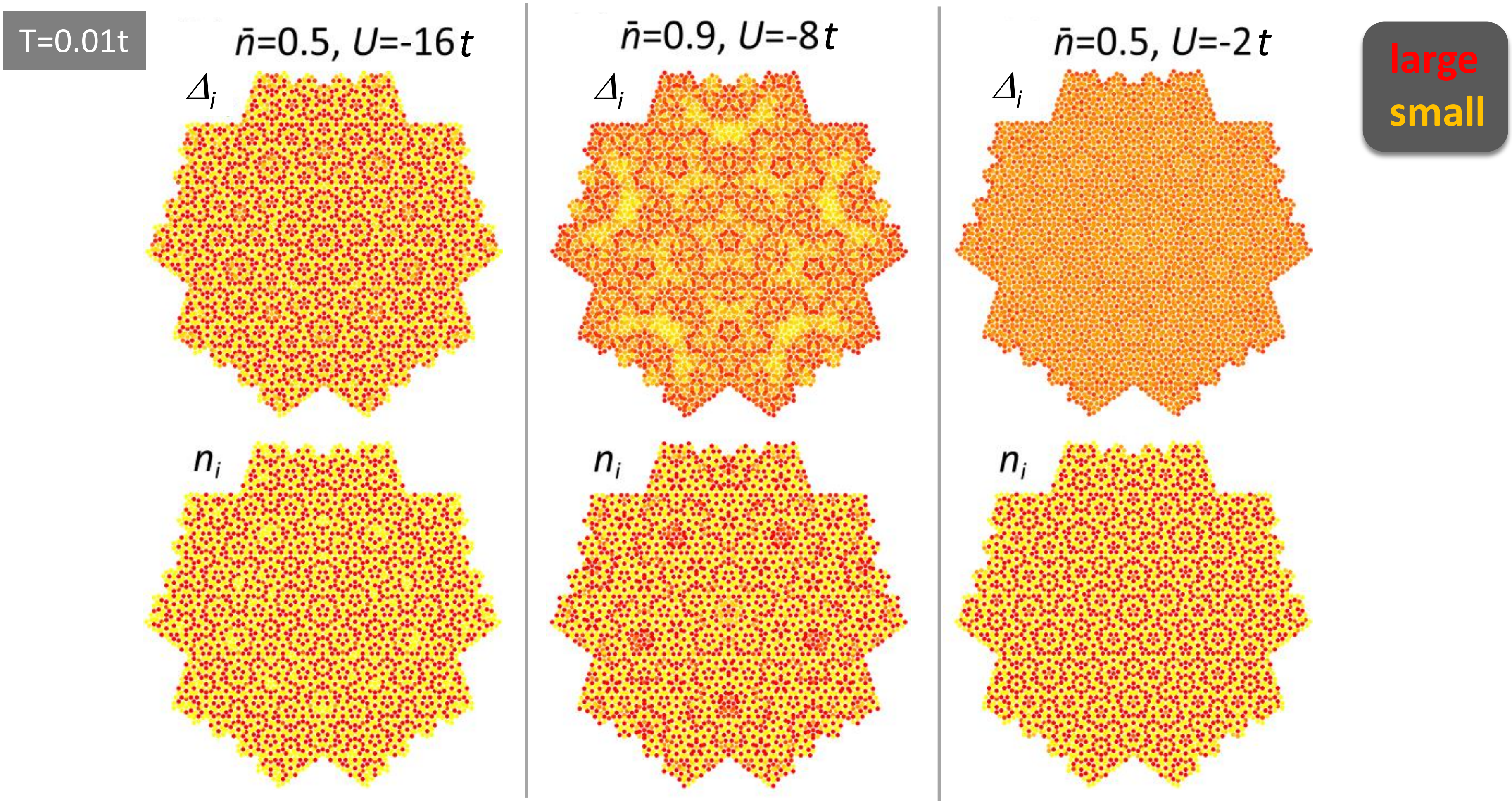
Local SC order parameter

RDMFT
N=4181 sites



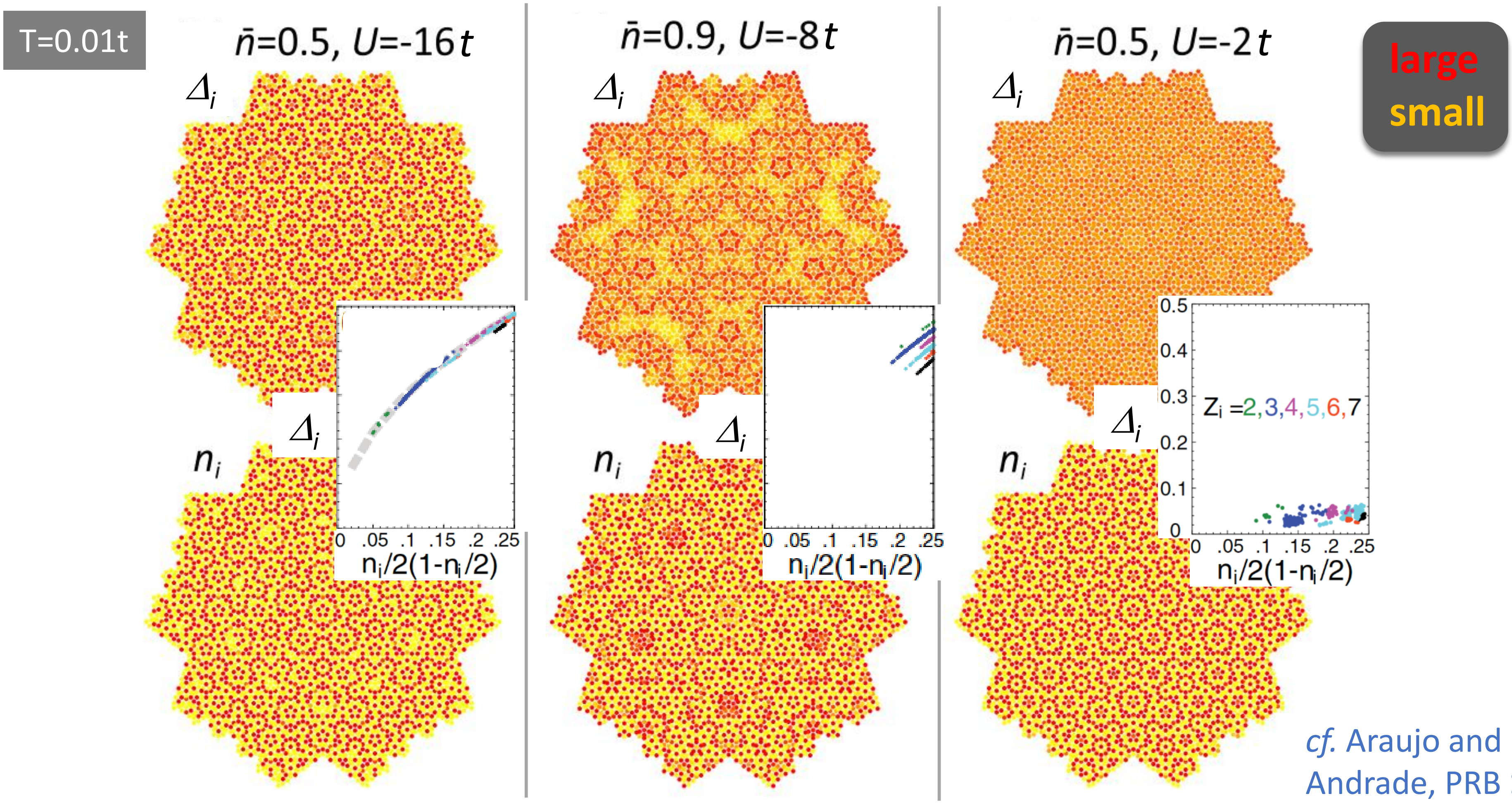
- Superconductivity occurs at low *T*.
- Transition occurs *simultaneously* at every sites.

Three different superconducting states



Δ_i	Order similar to n_i	Order different from n_i	No clear pattern

Three different superconducting states

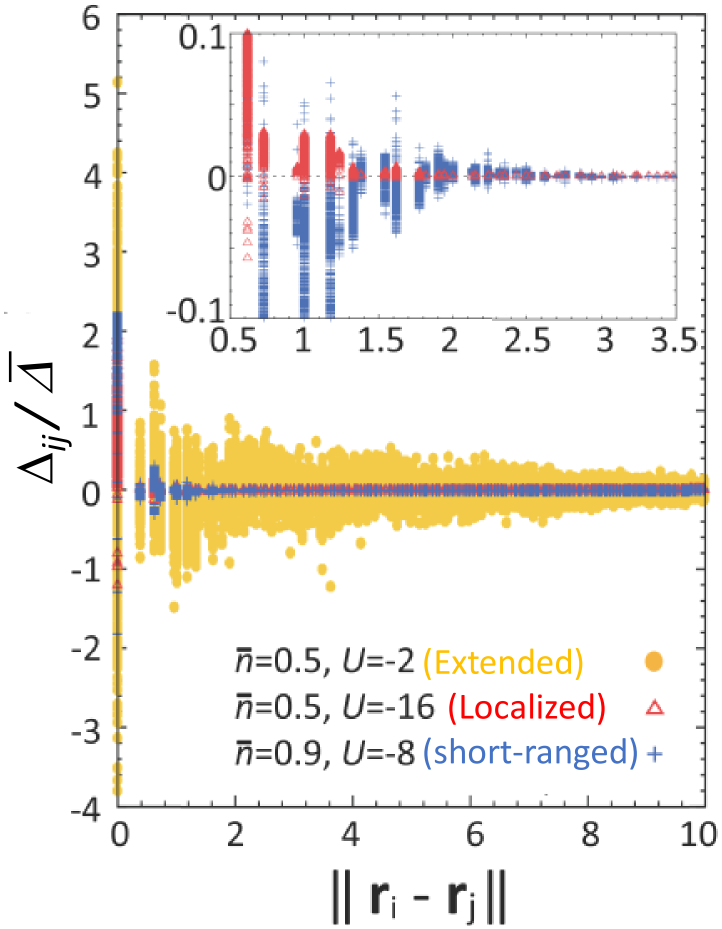


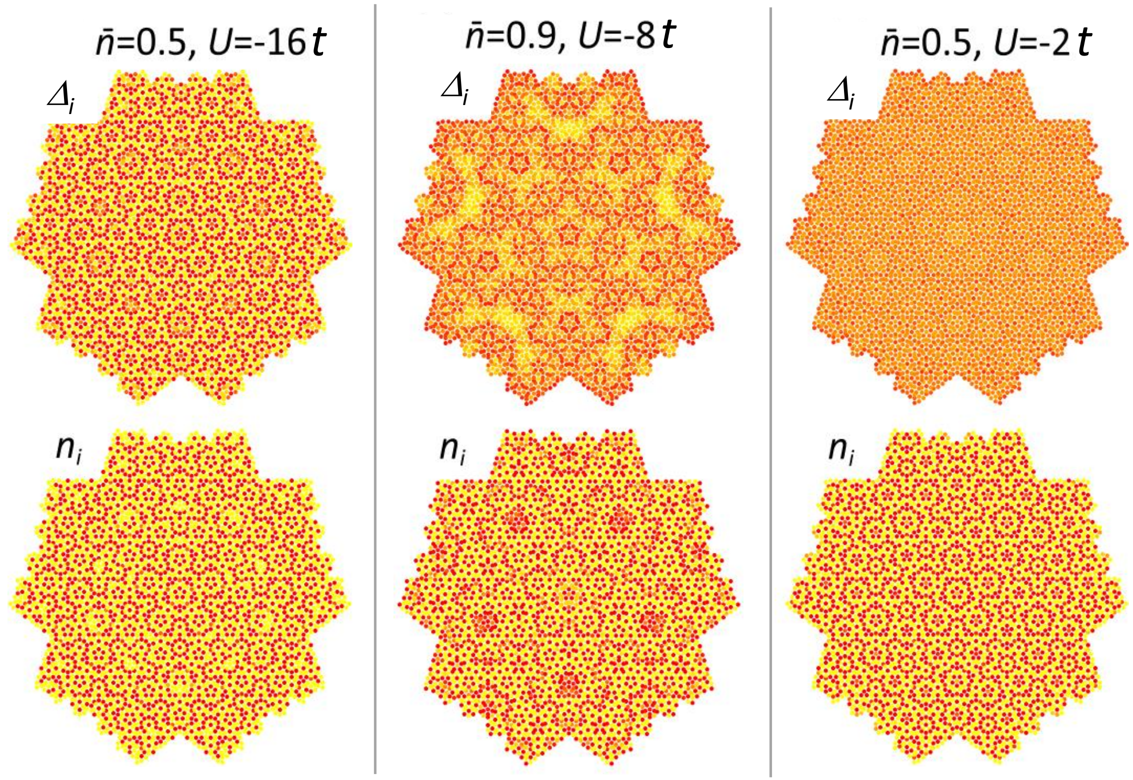
cf. Araujo and Andrade, PRB **100**, 014510 (2019)

Δ_i	Order similar to n_i	Order different from n_i	No clear pattern
	→ Determined by n_i	→ Determined by Z_i	→ Determined by LDOS(?)

Spatial extension of Cooper pairs

$\Delta_{ij}=\langle c_{i\uparrow}c_{j\downarrow}\rangle$: Off-site SC order parameter





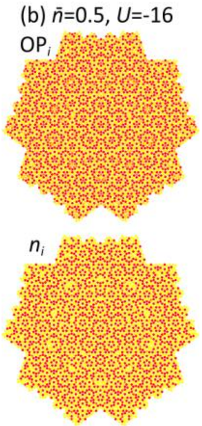
Red (n_i) **Blue (Z_i)** **Yellow (LDOS)**

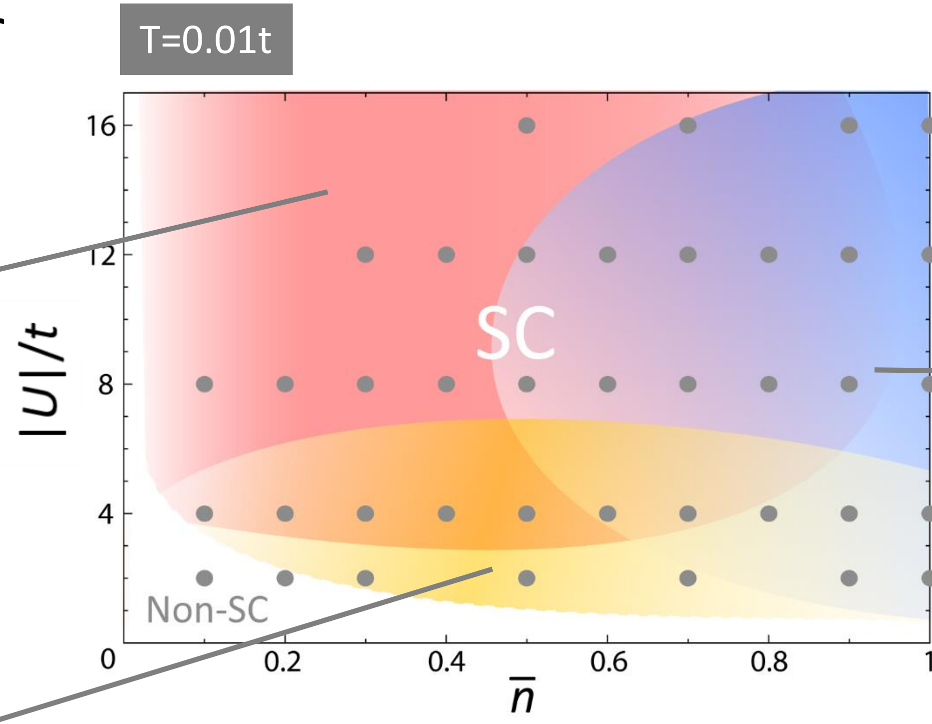
Cooper pairs are

Localized **Short-ranged** **Extended**

Crossover of three different SC states

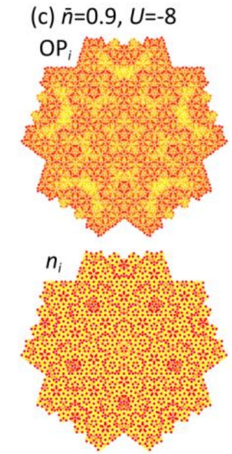
Localized pair



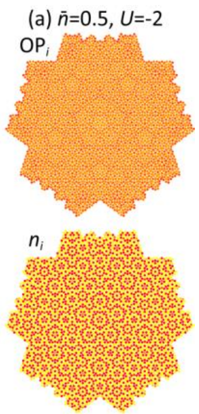


(Average el. density)

Short-ranged pair



Extended pair



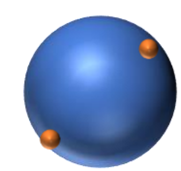
cf. Cooper instability in quasiperiodic system

Y. Zhang *et al.*, arXiv:2002.06485

$$|\Psi_A\rangle = \sum_{mn}^{\tilde{\varepsilon}_{m,n}>0} a_{mn}\hat{c}^\dagger_{m\uparrow}\hat{c}^\dagger_{n\downarrow}|\mathrm{FS}\rangle$$

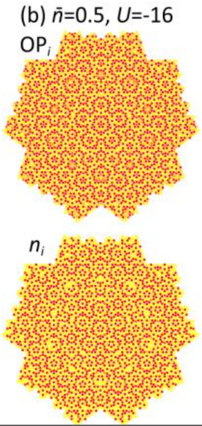
$\langle\Psi_A|\hat{H}|\Psi_A\rangle < 0$ for any U<0

$$\Delta \propto \exp\left[\frac{1}{\alpha U}\right]$$

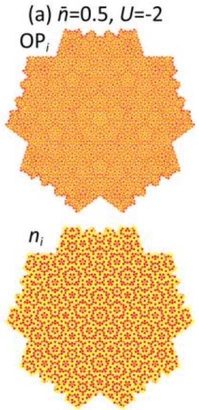


Crossover of three different SC states

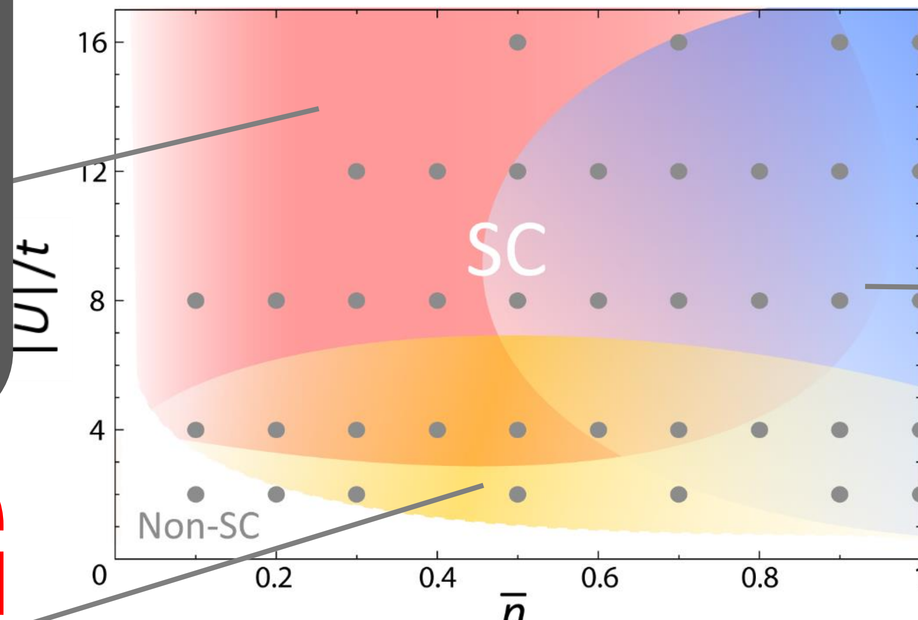
Localized pair



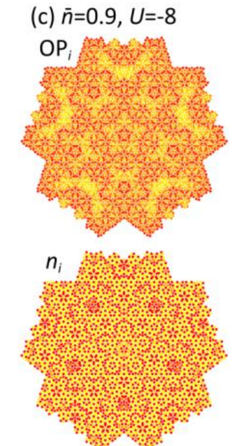
Extended pair



BEC: Lattice structure may be irrelevant



Short-ranged pair



Extended without Fermi surface!
What's happening?

SS, N. Takemori, A. Koga, and R. Arita, PRB **95**, 024509 (2017).

Unusual pairing in momentum space (1)

Cooper pair on periodic lattice: $c_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}$

➡ $c_{\mathbf{k}\uparrow}c_{\mathbf{k}'\downarrow}$ for aperiodic lattice

FT of relative coordinate *i-j*

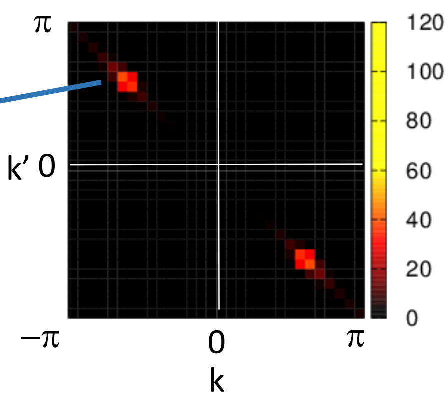
FT of *i* and *j, respectively*

Square lattice

$|\langle c_{\mathbf{k}\uparrow}c_{\mathbf{k}'\downarrow}\rangle|$ for k_x=k_y=k and k_x'=k_y'=k'

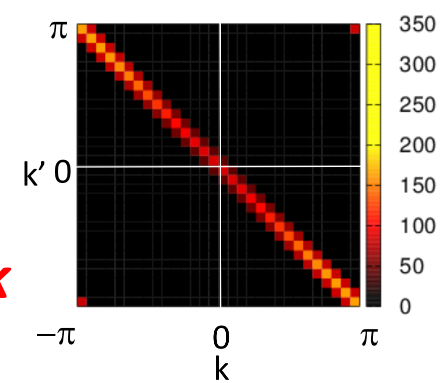
BCS **(U=-t, n=0.5)**

Fermi momentum



Finite only along k'=-k

BEC **(U=-16t, n=0.5)**

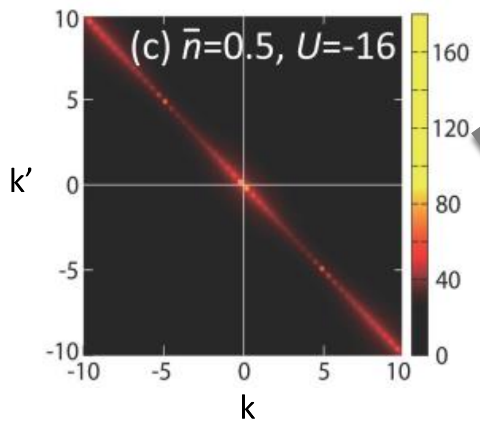


SS, N. Takemori, A. Koga, and R. Arita, PRB **95**, 024509 (2017).

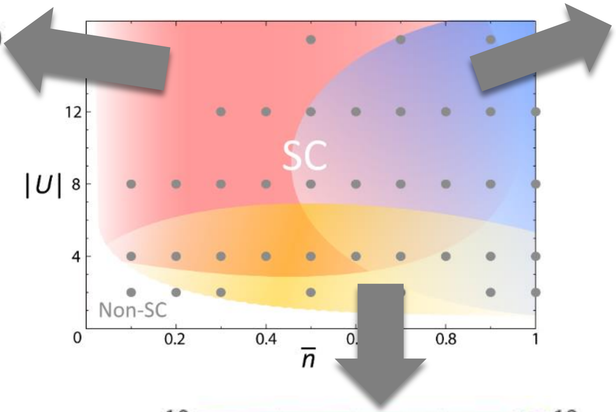
Unusual pairing in momentum space (2)

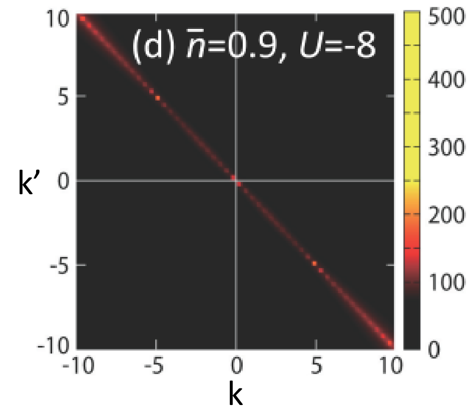
$|\langle c_{\mathbf{k}\uparrow} c_{\mathbf{k}'\downarrow} \rangle|$ for $k_x=k_y=k$ and $k_x'=k_y'=k'$

Penrose lattice

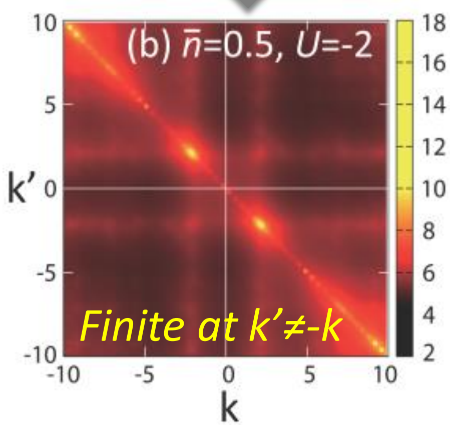


Localized pair
→BEC-like





Short-ranged pair
→BEC-like



Extended pair

Different from both
BCS and BEC characteristics

What about the property?

Universal properties in BCS theory

$$\frac{2E_g^0}{T_c} \cong 3.52$$

$E_g(T)$: Energy gap
$E_g^0 = E_g(0)$

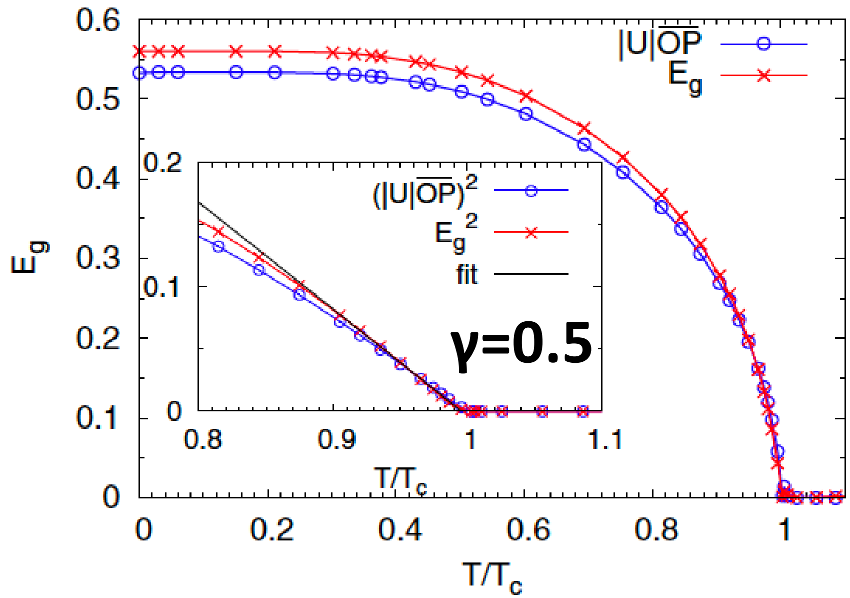
$$\frac{E_g(T)}{E_g^0} \cong A_1\left(1-\frac{T}{T_c}\right)^{\gamma}, A_1 \cong 1.74, \gamma = \frac{1}{2}$$

$\frac{\Delta C_e}{C_{en}} \cong 1.43$: Jump of specific heat

Does these relations hold in quasiperiodic SC?

The gap & T_c

BdG
U=-3t



cf. Amorphous SC

amorphous metal	T_c [K]	$2\Delta_0$ [meV]	$\frac{2\Delta_0}{kT_c}$	λ
Bi	6,1	2,42	4,60	2,2 - 2,46
Ga	8,4	3,32	4,60	1,94 - 2,25
$Sn_{0.9}Cu_{0.1}$	6,76	2,6	4,46	1,84
$Pb_{0.9}Cu_{0.1}$	6,5	2,66	4,75	2,0
$Pb_{0.75}Bi_{0.25}$	6,9	2,96	4,98	2,76
$In_{0.8}Sb_{0.2}$	5,6	2,13	4,40	1,69
$Tl_{0.9}Te_{0.1}$	4,2	1,67	4,6	1,70

Strong coupling SC

Bergmann, Phys. Rep. **27**, 159 (1976)

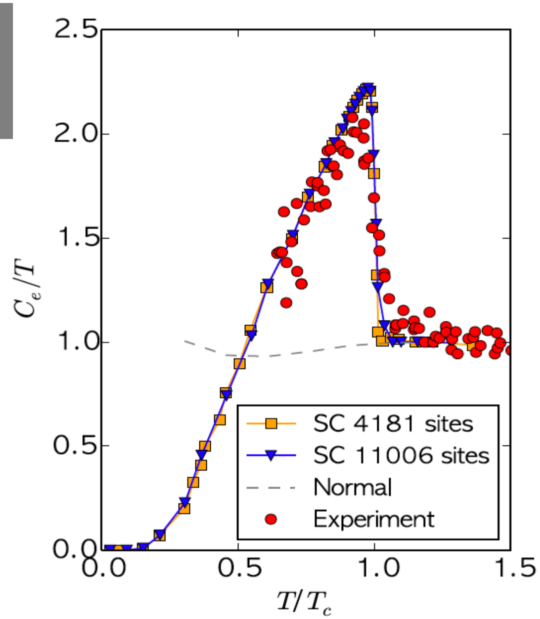
	Penrose				Square		BCS
	1591	4181	11 006	Ext	2500	10 000	
$\frac{2E_g^0}{T_c}$	3.35	3.38	3.38	3.38	3.46	3.45	3.52
A_1	1.61	1.63	1.69	1.70	1.70	1.70	1.74

$2E_g^0/T_c$: Small but substantial shift to a lower value ⬌ Amorphous SC

A_1: No significant change

Jump of specific heat

BdG
U=-3t



$$S = 2\sum_{\alpha}\left\{\ln(1+e^{-\beta E_\alpha}) + \frac{\beta E_\alpha}{e^{\beta E_\alpha}+1}\right\}$$

$$C_e = T\frac{dS}{dT}$$

Experiment: Kamiya *et al*., Nature Commun. **9**, 154 (2018)

	Penrose				Square		BCS
	1591	4181	11 006	Ext	2500	10 000	
$\frac{\Delta C}{C_{en}}$	1.13	1.21	1.21	1.21	1.40	1.39	1.43

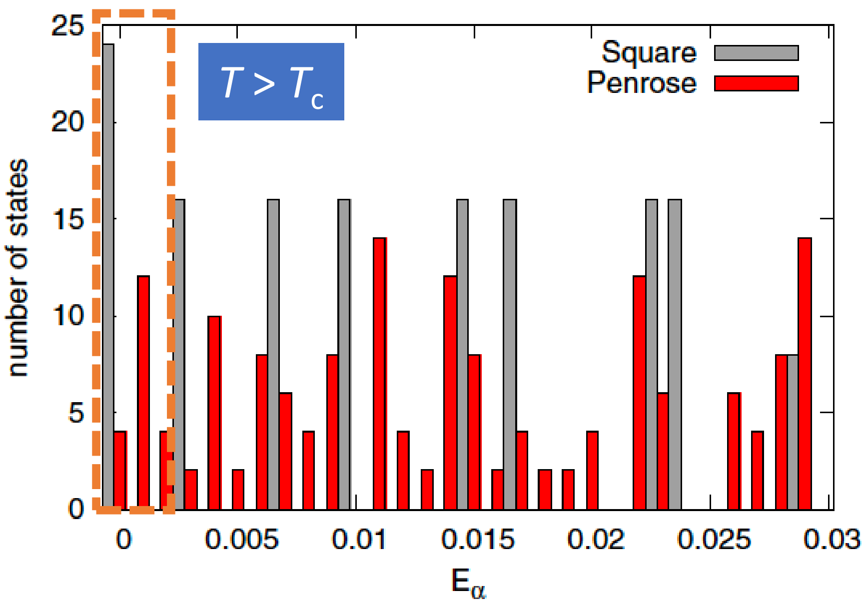
~15 % smaller Jump

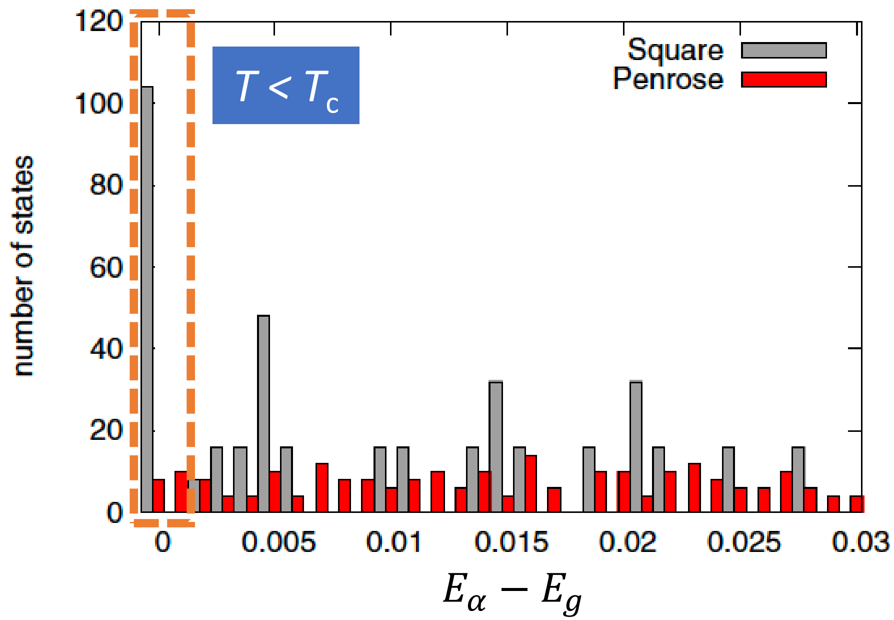
Absence of coherence peak

BdG
U=-3t

$$C_e = 2\beta \sum_{\alpha} \left(-\frac{\partial f(E_\alpha)}{\partial E_\alpha} \right) \left(E_\alpha^2 + \frac{\beta}{2} \frac{\partial E_\alpha^2}{\partial \beta} \right)$$

Only around E_F





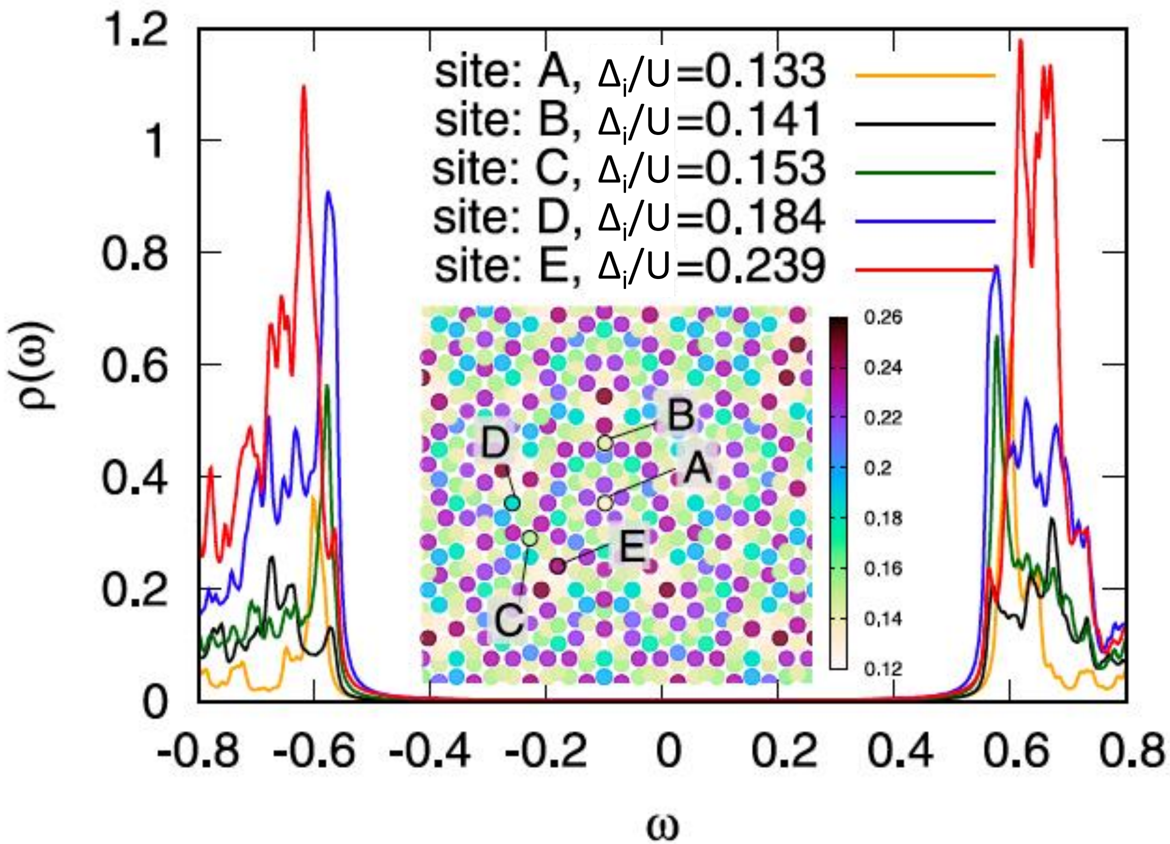
Absence of Fermi surface → Absence of coherence peak

→ Smaller jump

N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).

Local density of states

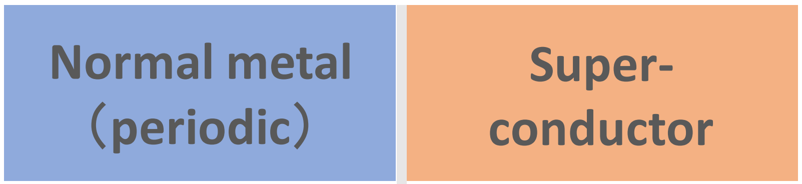
BdG
U=-3t



- The spectral gap is much more uniform than $\langle c_{i\uparrow}c_{i\downarrow}\rangle$.
- The weight of spectral peaks significantly depends on sites.

N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).

I-V characteristics (1)



$$I(V) \propto \int_{-\infty}^{\infty} \rho(E)[f(E) - f(E + eV)]dE$$

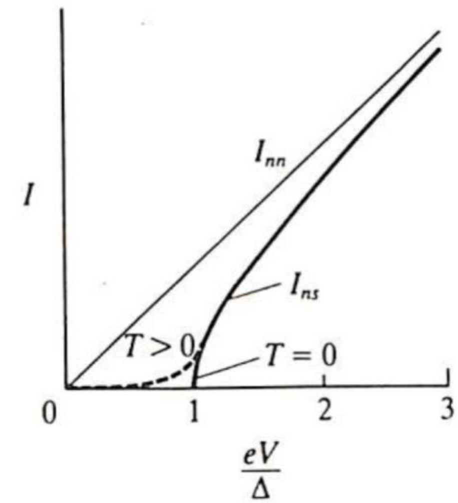
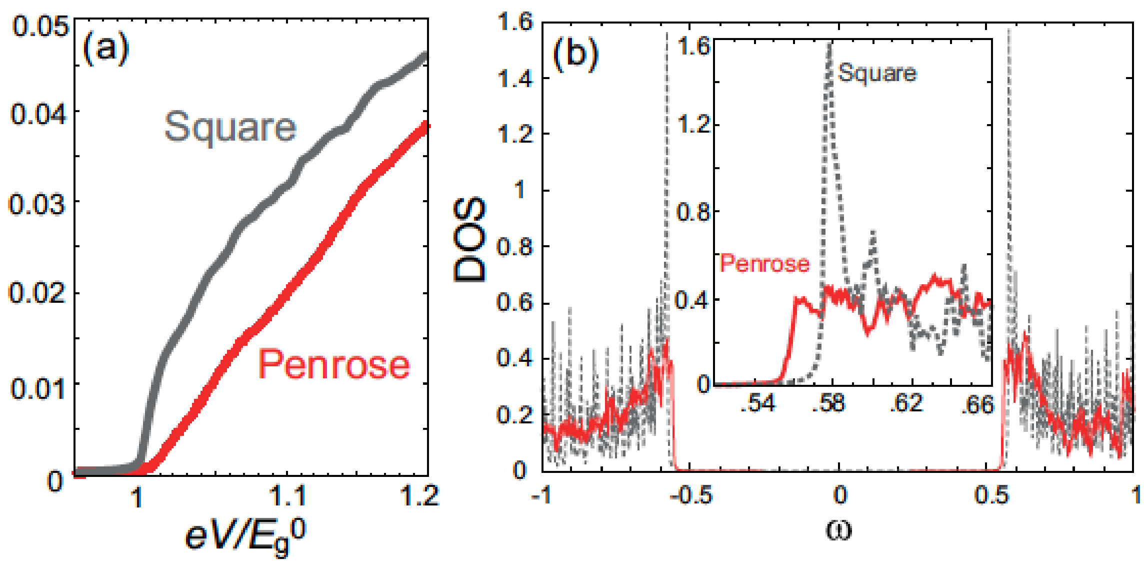


Fig. from M. Tinkham, INTRODUCTION TO SUPERCONDUCTIVITY

$T = 0$

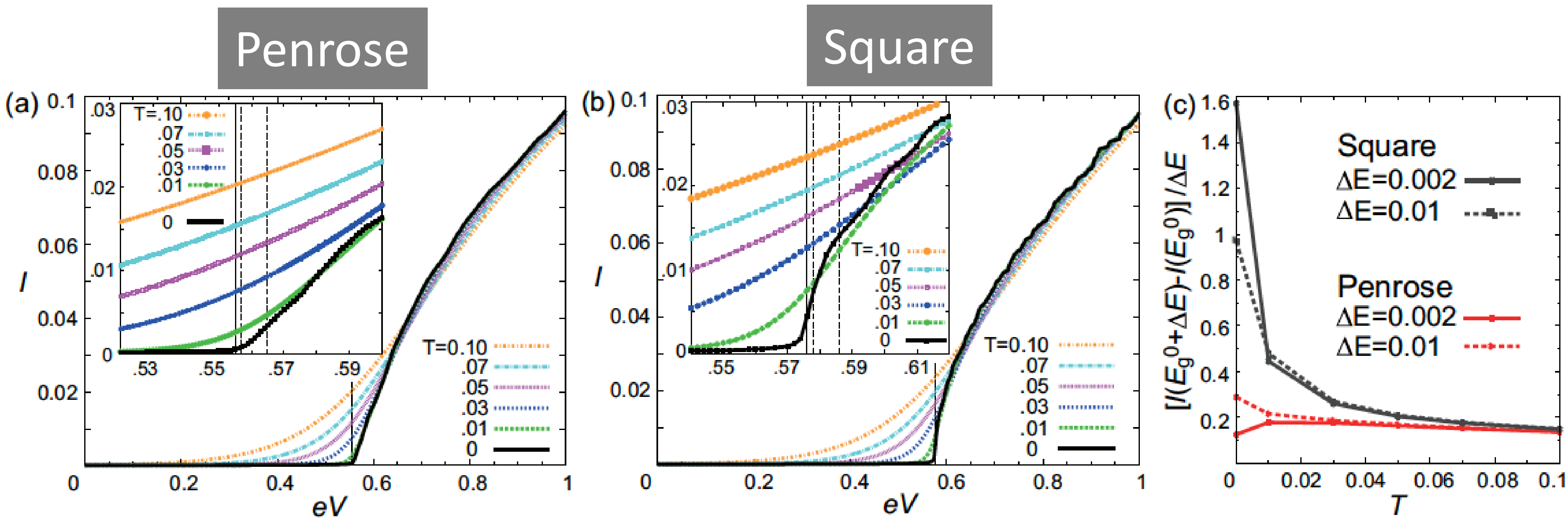


Finite gradient at the threshold voltage ⇐ Absence of coherence peak

N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).

I-V characteristics (2)

Any signature at finite T?



Weakly T-dependent slope signals quasiperiodic SC.

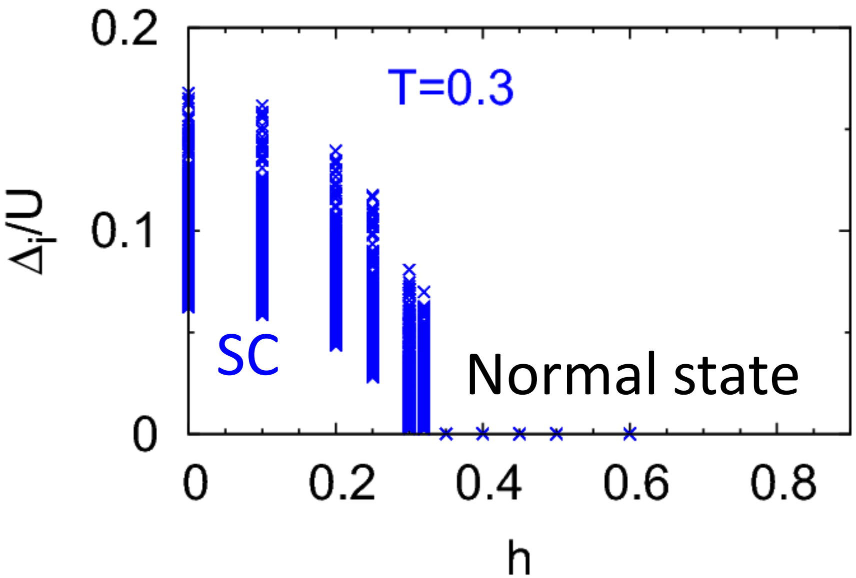
N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).

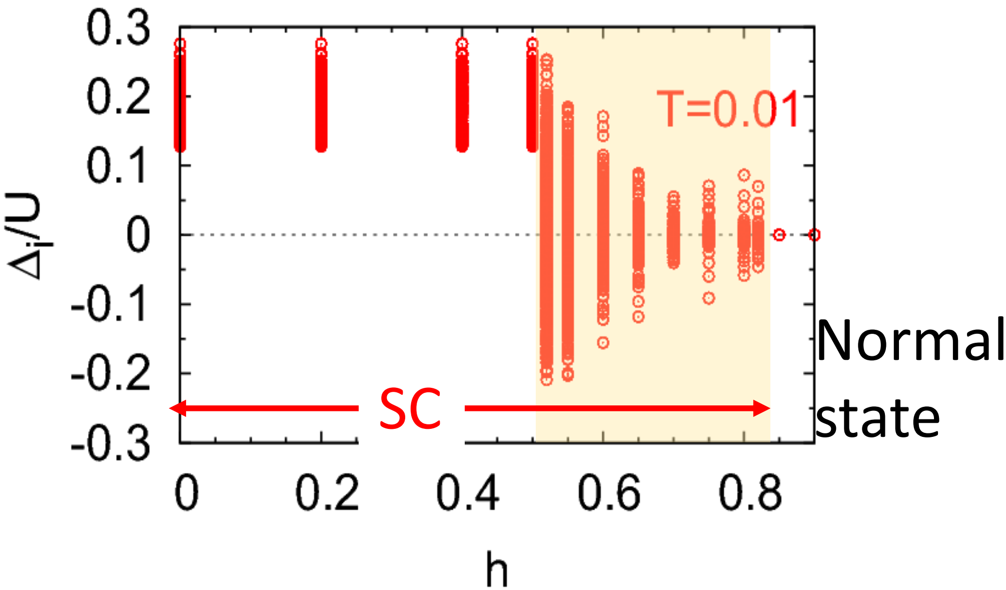
Effect of magnetic field

BdG
11006 sites
$\bar{n}$=0.5
U=-3t

Only Zeeman effect, no orbital motion : Magnetic field parallel to plane

T_c (h=0)=0.34



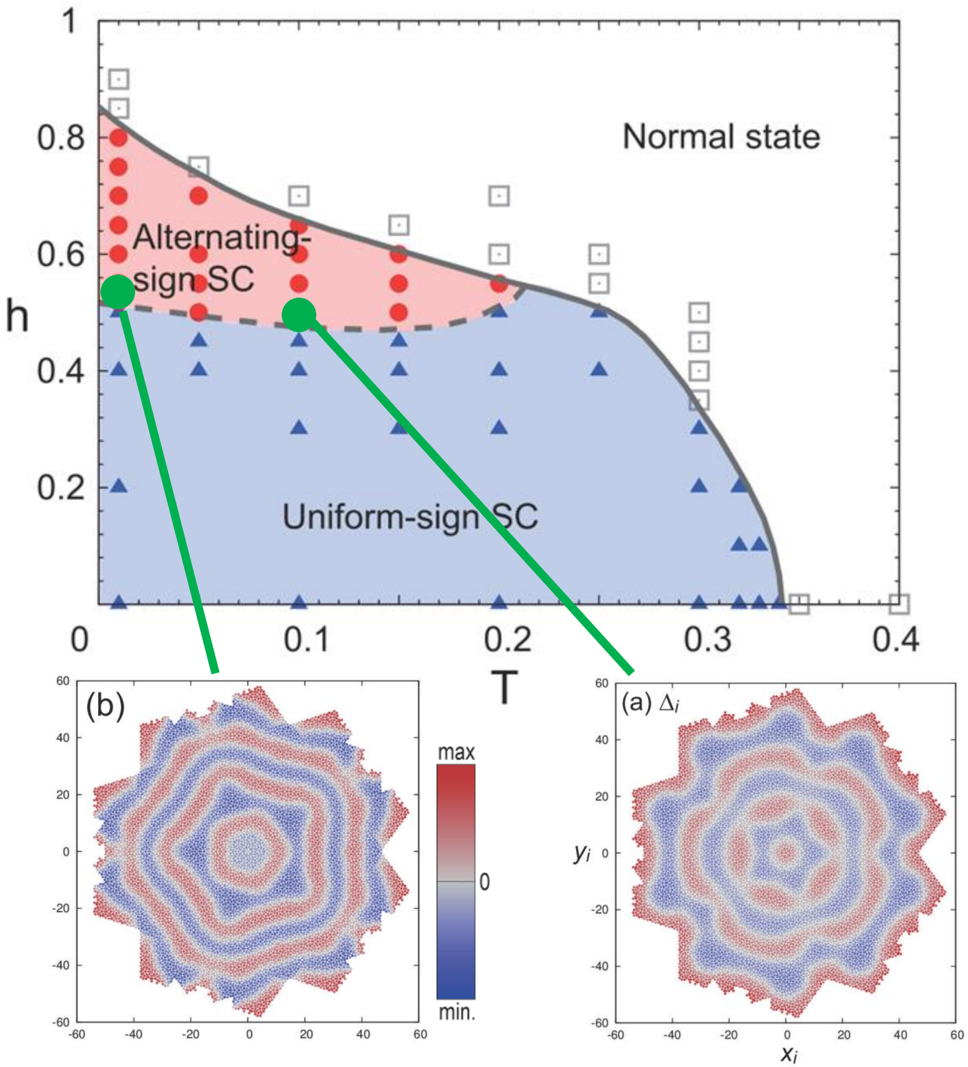


- 1st order transition.
- $\Delta_i \geqq 0$.

- Strange behavior before H_c .
- Both positive and negative Δ_i .

FFLO-like state in quasiperiodic systems

$\bar{n}$=0.5, U=-3



FFLO in periodic systems

Fulde and Ferrell, PR **135**, A550 (1964).
Larkin and Ovchinnikov, ZETF **47**, 1136 (1964).

$$\langle c_{\mathbf{k}+\mathbf{q}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$$

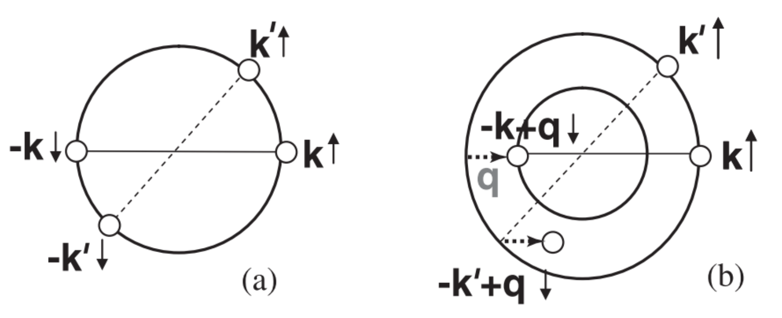


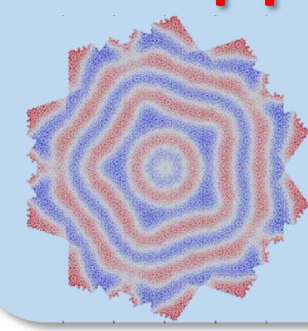
Fig. from Matsuda and Shimahara, JPSJ **76**, 051005 (2007).

Even without Fermi surface, the sign change occurs!

Impurities
(random potential)

Quasiperiodic potential

FFLO-like states



Electrons self-organize into a pattern compatible with the quasiperiodicity!

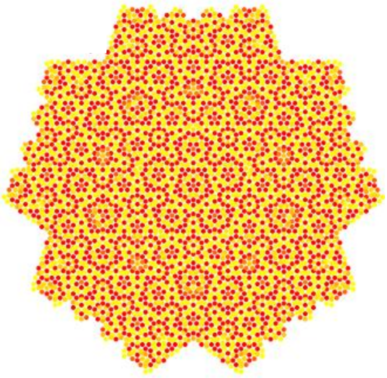
cf. Other exotic SCs:

- Anisotropic SC, spin-triplet SC — Y. Cao *et al.*, PRL **125**, 017002 (2020).
- Topological SC — I. C. Fulga *et al.*, PRL **116**, 257002 (2016). R. Ghadimi *et al.*, arXiv:2006.06952

Summary

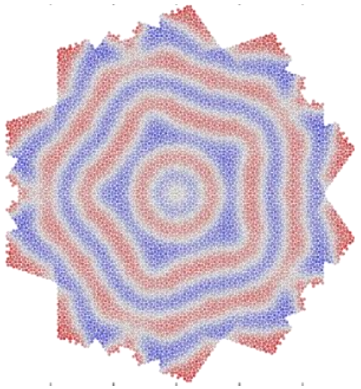
- Order parameter shows various spatial patterns, depending on the spatial extent of Cooper pairs.
- Weak-coupling SC with unusual extended Cooper pairs

SS, N. Takemori, A. Koga and R. Arita, PRB **95**, 024509 (2017).



- Deviation from the BCS universal properties

N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).



- FFLO-like SC under high magnetic field

SS and R. Arita, Phys. Rev. Research **1**, 022002(R) (2019).

QC can be a novel platform of exotic SC!